# Selected Solutions for Homework 5

Math 115

Oct. 11, 2004

1. (2.1.4) Consider the funtion$f(x)$ which is 1 for $x < 0$ and $-1$ for $x \geq 0$; this function is discontinuous at $x = 0$. Suppose $f$ has a tangent line (call it $l$) at $x = 0$. As $x \to 0^+$, the secant lines through $(x, f(x))$ and $(0, f(0))$ are all the horizontal line $y = -1$. However, as $x \to 0^-$, the slopes of the secant lines become unbounded. Therefore, you get different tangent lines if you approach $x = 0$ from the right or the left.

2. (2.1.6) The graph is a semincircle, so the tangent line at $x = 1$ is perdendicular to the radial line. Therefore the tangent line is the vertical line $x = 1$.

3. (2.1.13) We have $f(x) = x^3 - x$, and we want to compute the slopes of various secant lines.
   (a) $x = 1, 2$: then the line contains the points $(1, f(1)) = (1, 0)$ and $(2, f(2)) = (2, 6)$. So the slope of the line is $(6 - 0)/(2 - 1) = 6$.
   (b) $x = 2, 3$: then the line contains the points $(2, f(2)) = (2, 6)$ and $(3, f(3)) = (3, 24)$. So the slope of the line is $(24 - 6)/(3 - 2) = 18$.
   (c) $x = 3/2, 2$: the line contains $(3/2, f(3/2)) = (3/2, 15/8)$ and $(2, f(2)) = (2, 6)$. So the slope is $(6 - 15/8)/(2 - 3/2) = 33/4$.
   (d) $x = 2, 5/2$: the line contains $(2, f(2)) = (2, 6)$ and $(5/2, f(5/2)) = (5/2, 105/8)$. So the slope is $(105/8 - 6)/(5/2 - 2) = 57/4$.
   (e) $x = 1.9, 2$: the line contains $(1.9, f(1.9)) = (1.9, 4.959)$ and $(2, f(2)) = (2, 6)$. So the slope is $(6 - 4.959)/(2 - 1.9) = 10.41$.
   (f) $x = 2, 2.1$: the line contains $(2, f(2)) = (2, 6)$ and $(2.1, f(2.1)) = (2.1, 7.261)$. So the slope is $(7.261 - 6)/(2.1 - 2) = 12.61$.
   (g) We can compute the slope of the tangent line. Indeed, this slope is

$$\lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{(2+h)^3 - 2 - h - 6}{h} = \lim_{h\to 0} \frac{11h + 6h^2 + h^3}{h} = \lim_{h\to 0}(11 + 6h^2 + h^3) = 11.$$

4. (2.1.22) We have $f(x) = x^2 - 2$ and $a = 0$. For any value of $h \neq 0$, the secant line through $(0, f(0)) = (0, -2)$ and $(h, f(h)) = (h, h^2 - 2)$ is
$$y = hx - 2.$$
Letting $h \to 0$, we find the tangent line
$$y = -2.$$

5. (2.2.3) The derivative of a function is the slope of the linear function which is closest to the original function. In particular, if the deriviative is positive, this linear function is increasing, so the original function is also increasing. Conversely, if the deriviative is negative, then the linear function is decreasing, so the original function is also decreasing.

6. (2.2.6)
$$\lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{3(h+1)^2 + 1 - 4}{h} = \lim_{h\to 0} \frac{3h^2 + 6h}{h} = \lim_{h\to 0}(3h + 6) = 6.$$

7. (2.2.21–26) (21 – c), (22 – e), (23 – a), (24 – d), (25 – b), (26 – f)

8. (2.2.39) First note that
$$\lim_{x\to 0} x^{2/3} = 0,$$
so $f$ is continuous at $x = 0$. One can show this rigorously (with a $\delta$-$\epsilon$ proof) by taking $\delta = \epsilon^{3/2}$. However, as $x \to 0$ the secant lines through $(0, 0)$ and $(x, x^{2/3})$ get steeper and steeper. These lines approach the vertical line $x = 0$, and so the slopes become unbounded. In fact, the slope of the secant line through $(0, 0)$ and $(x, x^{2/3})$ is
$$m = \frac{x^{2/3} - 0}{x - 0} = x^{-1/3},$$
which diverges as $x \to \infty$. Thus $f'(0)$ does not exist.

9. (2.2.44)
$$D_+f(0) = \lim_{h\to 0^+} \frac{f(h) - f(0)}{h} = \lim_{h\to 0^+} \frac{h^3}{h} = \lim_{h\to 0^+} h^2 = 0.$$
$$D_-f(0) = \lim_{h\to 0^-} \frac{f(h) - f(0)}{h} = \lim_{h\to 0^-} \frac{h^2}{h} = \lim_{h\to 0^-} h = 0.$$