Selected Solutions for Homework 2

Math 115

Sept. 15, 2004

Note on grading: I graded problems 1.1.5, 1.1.18, 1.1.30, 1.2.2 and 1.2.14; two points per problem.

1. (Problem 1.1.18) We want to compute
$$\lim_{x\to -1} \frac{|x+1|}{x^2-1}.$$
First let $f(x) := (|x+1|)/(x^2-1)$. For $x > -1$, we have $|x+1| = x+1$, and so
$$f(x) = \frac{x+1}{x^2-1} = \frac{x+1}{(x+1)(x-1)} = \frac{1}{x-1}.$$
Thus, as $x \to -1^+$, the function $f(x)$ approaches
$$f(x) \to \frac{1}{-1-1} = -\frac{1}{2}.$$
This is because both the numerator and demoniator are polynomials, as the demoninator is non-zero at $x = -1$. However, for $x < -1$, we have $|x+1| = -x-1$, so this time (for $x < -1$)
$$f(x) = \frac{-x-1}{(x+1)(x-1)} = -\frac{1}{x-1}$$
and as $x \to -1^-$
$$f(x) \to -\frac{1}{-1-1} = \frac{1}{2}.$$
Thus the righthand and lefthand limits disagree, and so the limit does not exist.

2. (Problem 1.1.30)We can define
$$f(x) = \begin{cases} 1 & x \leq 1 \\ 3x > 1. \end{cases}$$
This is one possible function which fits the requirements; there are many others. The graph looks like two horizontal rays. The ray going to the left has height 1 and the ray going to the right has height 3.

3. (Problem 1.2.2) Suppose one can draw the graph of the function $f(x)$ without lifting one's pen. Then for any a, as the values of x approach a, the values of $f(x)$ must approach $f(a)$ this is because the values $f(x)$ are the heights of the points on the graph, for corresponding horizontal positions x. In other words, because the graph has no skips or breaks, the heights of the points $(x, f(x))$ have to be close to the height of $(a, f(a))$ as x gets close to a. This is the same thing as
$$\lim_{x\to a} f(x) = f(a).$$

4. (Problem 1.2.14)
$$\lim_{x\to 0} \frac{x^2+x}{x^2-3x} = \lim_{x\to 0} \frac{x(x+1)}{x(x-3)} = \lim_{x\to 0} \frac{x+1}{x-3} = -\frac{1}{3}.$$
We can carry out the final step because the numerator and denominator are both polynomials and the denominator is non-zero at $x = 0$.