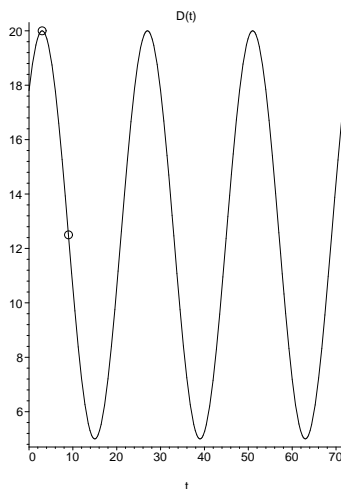


Practice Problems
Math 1060
November 23, 2002

- Suppose the shadow of a tree is 20 feet long when the angle of elevation the sun makes with the ground is $\pi/6$. Find the height of the tree.
- Suppose a plane is flying at 600 miles per hour and a height of 20 miles. A direct line from a satellite dish on the ground to the plane makes an angle of $\pi/12$ with the ground.
 - What is the horizontal distance from the plane to the satellite dish?
 - What is the distance between the plane and the satellite dish?
 - How much time elapses before the plane passes over the satellite dish, assuming that it flies at the same height and speed, in a straight line directly over the satellite dish?
- If $\sin \theta = -1/\sqrt{2}$ and $\cos \theta > 0$ find
 - $\cos \theta$
 - $\csc \theta$
 - $\tan \theta$
- If $\tan \theta = 1$ and $\cos \theta < 0$ find
 - $\cos \theta$
 - $\sin \theta$
 - $\csc \theta$
- If $\cot \theta = -\sqrt{3}$ and $\sin \theta < 0$ find
 - $\sin \theta$
 - $\cos \theta$
 - $\sec \theta$
- Consider the function $F(t) = 1 + 2 \cos(2(t - \pi/3))$
 - What is the average value of F ?
 - What is the amplitude of F ?
 - What is the period of F ?
 - What is the phase shift of F ?
 - Sketch a graphs of 3 periods of F .
- Consider the function $F(t) = 3 \tan(\pi(t - 1/2)) - 2$.
 - What is the average value of F ?
 - What is the amplitude of F ?
 - What is the period of F ?
 - What is the phase shift of F ?
 - Sketch a graphs of 3 periods of F .
- Suppose the temperature during an average July day has a highest value of 100, achieved at 3PM and that the lowest temperature is 70, achieved at 3AM. Let $T(t)$ be the function which measures the temperature T at time t , where t is the number of hours after midnight.
 - What is the period of T ?
 - What is the average temperature and when does it occur?
 - Sketch the function $T(t)$ for two periods.
 - Find a formula for T of the form $T(t) = a \cos(b(t - c)) + d$.
 - Find a formula for T of the form $T(t) = A \sin(B(t - C)) + D$.

- (f) How many hours per day is the temperature below 65?
 (g) How many hours per day is the temperature above 80?
9. Suppose the depth of the water at the end of a pier is given by the graph below. The two marked points are (3, 20) and (9, 12.5).



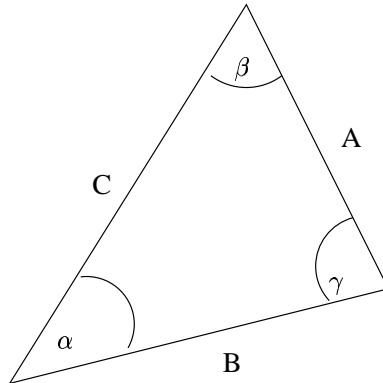
- (a) What is the period of D ?
 (b) What is the lowest depth and when does it occur?
 (c) Find a formula for D of the form $D(t) = a \cos(b(t - c)) + d$.
 (d) Find a formula for D of the form $D(t) = A \sin(B(t - C)) + D$.
 (e) How many hours per day is the depth below 22?
 (f) How many hours per day is the depth above 15?
10. Find all the solutions to the equation $\sin(2\theta - \pi/3) = 1/2$ with
- (a) $-\pi/2 \leq \theta < \pi/2$
 (b) $3\pi/2 \leq \theta < 5\pi/2$
 (c) $-2\pi \leq \theta < -\pi$
11. Find all the solutions to the equation $\tan(3\theta - \pi/2) = 1$ with
- (a) $\pi/2 \leq \theta < 3\pi/3$
 (b) $-3\pi/2 \leq \theta < -\pi/2$
 (c) $\pi \leq \theta < 2\pi$
12. Find all the solutions to the equation $\cos^2 \theta + 2 \sin \theta = 1$ with
- (a) $0 \leq \theta < \pi$
 (b) $-\pi/3 \leq \theta < 2\pi/3$
 (c) $\pi/2 \leq \theta < 3\pi/2$
13. Find all the solutions to the equation $\cos^2 \theta - 2 \sin \theta = 0$ with
- (a) $-3\pi/2 \leq \theta < -\pi/2$
 (b) $3\pi/2 \leq \theta < 5\pi/2$
 (c) $\pi \leq \theta < 2\pi$
14. Using the identities $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$ and $\cos^2 \theta + \sin^2 \theta = 1$ show

$$\cos(\theta/2) = \sqrt{\frac{1 + \cos \theta}{2}}.$$

15. Suppose $\cos(u) = \sqrt{3}/2$ and $\sin(v) = -1/\sqrt{2}$. If u and v are both in the fourth quadrant, find

- (a) $\cos(u + v)$
- (b) $\sin(u - v)$
- (c) $\tan(u + v)$
- (d) $\cos(2u)$
- (e) $\sin(2v)$
- (f) $\sin(v/2)$

16. Consider the triangle drawn below.



In each case where you are given some of the information regarding the triangle (e.g. two angles and a side length) find the remaining side lengths and angles (when possible; if no solution exists, explain why and if multiple solutions exist find all of them). (Caution: the figure may not be drawn to scale. Also, it may help to reorient the triangle.)

- (a) $\alpha = \pi/3, \beta = \pi/4, C = 10$
- (b) $\alpha = \pi/6, \beta = \pi/2, A = 5$
- (c) $\alpha = \pi/3, A = 4, B = 3$
- (d) $A = 3, B = 5, C = 7$
- (e) $A = 3, B = 5, \gamma = \pi/3$
- (f) $\alpha = \pi/4, A = 3, B = 4$.

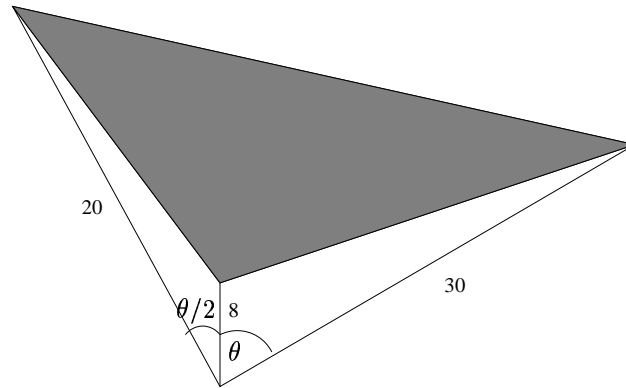
17. Find the area of the triangles with the following properties. Sides and angles are labeled as in the previous problem.

- (a) base of 10 and height of 5
- (b) $A = 10, B = 12$ and $\gamma = \pi/4$
- (c) $A = 10, B = 8$ and $\beta = \pi/3$
- (d) $A = 10, B = 8$ and $C = 6$
- (e) $A = 10, \alpha = \pi/3$ and $\beta = \pi/4$
- (f) $A = 10, \gamma = \pi/3$ and $\beta = \pi/4$.
- (g) $A = 10, B = 8$ and $C = 12$
- (h) $A = 10, \beta = \pi/4$ and $C = 8$

18. Show that the area of the parallelogram shown below is $AB \sin \theta$.



19. Suppose you know two side lengths and an angle of a triangle. Can you determine the area of the triangle? Justify your answer either way (i.e. either explain why you can find the area or find a counterexample where you can't).
20. Suppose you are given three pieces of information regarding a triangle (e.g. two angles and a side length). What is the maximum number of triangles which satisfy the conditions you're given (i.e. have the same information as you're given)? Explain your answer.
21. Consider the three triangles drawn below.



- (a) Find the area of the shaded triangle as a function of θ .
- (b) For which values of θ is this area positive?
22. Use the law of cosines ($A^2 = B^2 + C^2 - 2BC \cos \alpha$, where the triangle is again labeled as in problem 16) to show
- $$\frac{1}{2}BC(1 - \cos \alpha) = \frac{A + B - C}{2} \cdot \frac{A - B + C}{2}.$$
23. A fire at location F is spotted from two fire stations, A and B , which are 10.3 miles apart. If the angle ABF is $\pi/12$ and the angle BAF is $\pi/3$ find the distance from the fire to each of the fire stations.
24. A regular pentagon is a five sided polygon such that all the interior angles are equal. Find the perimeter (i.e. the length of the boundary curve) of a regular pentagon inscribed in a circle of radius 5 (see the figure below).

