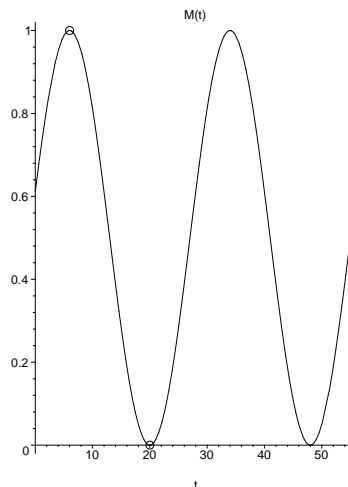


Practice Problems
Math 1060
October 29, 2002

1. Consider the function $F(t) = 1 - \cos(\pi/2(t - 3))$.
 - (a) What is the period of F ?
 - (b) What is the amplitude of F ?
 - (c) What is the average value of F ?
 - (d) What is the phase shift of F ?
 - (e) Sketch the graph of F for three periods.
2. Consider the function $F(t) = \frac{3}{2} \sin(3(t - \pi/5)) - 3$.
 - (a) What is the period of F ?
 - (b) What is the amplitude of F ?
 - (c) What is the average value of F ?
 - (d) What is the phase shift of F ?
 - (e) Sketch the graph of F for three periods.
3. Consider the function $F(t) = 2 \tan(2(t - \pi/3)) + 1$.
 - (a) What is the period of F ?
 - (b) What is the amplitude of F ?
 - (c) What is the average value of F ?
 - (d) What is the phase shift of F ?
 - (e) Sketch the graph of F for three periods.
4. Suppose the depth of the water at the end of a dock varies periodically, with one period per 24 hours. Suppose also that the peak depth of 25 feet occurs at 3AM and that the average depth is 15 feet. Let $D(t)$ represent the depth of the water at time t , where t measures hours after midnight.
 - (a) What is the lowest depth of the water and when does it occur?
 - (b) When does the depth achieve its average value?
 - (c) Sketch the graph of D for three periods.
 - (d) Find a formula for $D(t)$ of the form $D(t) = a \cos(b(t - c)) + d$.
 - (e) Find a formula for $D(t)$ of the form $D(t) = A \sin(B(t - C)) + D$. (There are at least two ways to do this.)
 - (f) Suppose a boat need a depth of at least 10 feet in order to moor to the dock. When can the boat moor?
 - (g) Suppose there are some rocks which are exposed when the depth of the water is less than 7 feet. When are the rocks exposed?
 - (h) Suppose there is some buried treasure which is exposed when the depth of the water is less than 3 feet. When is the buried treasure exposed?
5. Suppose the phase of the moon is given by the following graph. Let $M(t)$ be the percentage of the moon illuminated on day t , where t represents days after Jan. 1.



The two marked points are (6, 1) and (20, 0).

- (a) What is the period of M ?
- (b) What is the amplitude of M ?
- (c) What is the average value of M ?
- (d) What is the phase shift of M ?
- (e) Find a formula for M of the form $M(t) = a \cos(b(t - c)) + d$.
- (f) Find a formula for M of the form $M(t) = A \sin(B(t - C)) + D$.
- (g) How many days per cycle is the percentage of the moon illuminated at least $3/4$?
- (h) How many days per cycle is the percentage of the moon illuminated at least $1/4$?

6. Find all solutions θ to the equation $\tan(2\theta - \pi/3) = 1/3$ where

- (a) $-\pi/2 \leq \theta < \pi/2$
- (b) $\pi/2 \leq \theta < 3\pi/2$
- (c) $-2\pi \leq \theta < -\pi$

7. Find all solutions θ to the equation $\sin(3\theta - \pi/4) = 1/5$ where

- (a) $\pi/2 \leq \theta < \pi/2$
- (b) $-3\pi/2 \leq \theta < -\pi/2$
- (c) $\pi \leq \theta < 2\pi$

8. Find all solutions θ to the equation $\sin^2 \theta + \frac{1}{3} \cos \theta = 1$ where

- (a) $0 \leq \theta < 2\pi$
- (b) $-2\pi \leq \theta < -\pi$
- (c) $\pi/2 \leq \theta < 3\pi/2$

9. Find all the solutions θ to the equation $\cos^2 \theta - \frac{1}{4} \sin \theta = 0$ where

- (a) $0 \leq \theta < 2\pi$
- (b) $-\pi \leq \theta < \pi$
- (c) $-\pi/2 \leq \theta < 3\pi/2$

10. Simplify the expression $\frac{\sin \theta}{1 + \cos \theta} + \cot \theta$ by adding the two quantities and then manipulating the result.

11. Verify that $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = \frac{2}{\cos^2 \theta}$.

12. Starting with the identities $\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi$ and $\sin(\theta + \phi) = \cos \theta \sin \phi + \cos \phi \sin \theta$, derive a formula for $\tan(\theta + \phi)$.