Problem 1) Consider $f(x) = e^x$ on $[0, 1]$. Write the Riemann sum $R_n$ using the midpoint rule (and $n$ subintervals of equal length). As a first step, you should find $x_i^*$ in terms of $i$ and $n$.

Problem 2) Evaluate the integral $\int_{-1}^{1} \sin(x)\,dx$. Explain. (Hint: no computation needed here).

Problem 3) Consider the expression
\[
\lim_{n \to \infty} \sum_{i=1}^{n} \frac{2\sqrt{4 - (\frac{2i}{n})^2}}{n}.
\]
(a) Express this as a definite integral.
(b) Using geometric considerations, find the above limit.

Problem 4) We propose an alternative proof of the formula
\[
\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.
\]
So suppose you don’t know this formula. We will show how to recover it. Let $S_n$ denote the value of $\sum_{i=1}^{n} i$ and let $F_n$ denote the value of $\sum_{i=1}^{n} i^2$.
(a) Explain in few words why $\sum_{i=1}^{n} (i - 1)^2 = F_n - n^2$.
(b) By expanding $(i - 1)^2$ and using properties of the sum, write $\sum_{i=1}^{n} (i - 1)^2$ in terms of $F_n$, $S_n$ and $n$.
(c) Using (a) and (b), recover the formula for $S_n$.

Remark: this trick also works for proving the other sum formulas, for instance $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Don’t forget to do the WebAssign Homework, due September 10th at 11:59 pm.

Some practice problems that you should try (do not submit them): Section 5.1: 19, 21, 23, 25. Section 5.2: 5, 9, 11, 19, 27, 37, 39, 49, 55, 57, 71.