MATH 227Q. Fall 2006.
Prof. Olshevsky's section

Review sheet for the first exam.

The exam will cover the material from Chapter 1 that we have discussed in class and studied in recommended exercises and in homework problems.

The following list points out the most important definitions, theorems and skills. It was written just to highlight the **main points**, and is not complete. In other words the actual exam may include any of the other results from Chapter 1 that we have studied. Still, I hope that even an incomplete list highlighting just the main points can be very useful while you will be preparing for the first exam.

Definitions .

The definition of the reduced row echelon form

The definition of $A\mathbf{x}$ in both words and symbols.

$Span\{\mathbf{v}\}$, $Span\{\mathbf{u}, \mathbf{v}\}$ and geometric interpretation in $\mathbb{R}^2$ or $\mathbb{R}^2$.

$Span\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}\}$.

Linearly dependent and linearly independent.

Linear Transformation.

One-to-one and onto (for linear transformation).

Matrix-Vector product.

Theorems .

Theorem 1 (Uniqueness of the reduced Echelon Form).

Theorem 2 (Existence and Uniqueness).

Theorem 3 (Matrix Equation, vector equation).

Theorem 4 (When do the columns of A span $\mathbb{R}^n$).

Theorem 5 (Properties of Matrix-Vector product).

Theorems 7, 8, 9 (Properties of linearly independent sets). Know well the proofs of theorems 8 and 9.

Theorems 11 and 12 (one-to-one and onto linear transformations).

Important Skills .

Determine when a system is consistent. Write the general solution in parametric vector form.

Determine values of parameters that make the system consistent or make the solution unique.

Describe existence or uniqueness of solutions in terms of pivot positions.

Determine when a homogeneous system has a nontrivial solution.

Determine when a vector is in the subset spanned by specified vectors.

Exhibit a vector as a linear combination of specified vectors.

Determine whether the columns of an $m \times n$ matrix span $\mathbb{R}^m$.

Determine whether the columns are linearly independent.

Write the parametric equation of a line through $\mathbf{p}$ in the direction parallel to $\mathbf{a}$.

Write the parametric equation of a line through the points $\mathbf{p}$ and $\mathbf{q}$.

Write an equation of a line or a plane in the parametric vector form.

Use linearity of matrix vector multiplication to compute $A(\mathbf{u}+\mathbf{v})$ or $A(c\mathbf{u})$.

Determine whether a set of vectors is linearly independent. Know several methods that can sometimes produce an answer ”by inspection”, i.e., without much calculation.

Determine whether a specified vector is in the range of a linear transformation.

Find the standard matrix of a linear transformation.

Determine whether a linear transformation $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is (a) one-to-one, or (b) maps $\mathbb{R}^n$ onto $\mathbb{R}^m$.