Name:______________________________ Section:__________

1. Given $\begin{array}{rcrcrcl} 2x_1 & + & x_2 & + & 5x_3 & = & b_1 \\ x_1 & - & x_2 & + & x_3 & = & b_2 \\ x_1 & & & + & 2x_3 & = & b_3 \end{array}$ and $A = \begin{bmatrix} 2 & 1 & 5 \\ 1 & -1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$.

(a) Compute all solutions to the above linear system by reducing its associated augmented matrix to row echelon form. Describe the set of all b_1, b_2, b_3 for which the system is consistent.

(b) Use your work in (a) to find the span$\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} \right\}$.

(c) Use your work in part (a) to determine if the columns of the matrix A are linearly independent. Explain your reasoning.

(d) Can the vector $\begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$ be written as a linear combination of the columns of A? If so, show the linear combination. If not, explain why it is not possible.

2. Let $T : R^3 \longrightarrow R^3$ be the linear transformation defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} x_1 + x_3 \\ 2x_1 + x_2 \\ x_2 - x_3 \end{bmatrix}$.

(a) Find the matrix A associated with T (that is, $T(\vec{x}) = A\vec{x}$).

(b) Is T one-to-one? (You may use any property of the Inverse Matrix Theorem to decide this but you need to explain and justify your reasoning.)

(c) Does T map R^3 onto R^3? Explain and justify your reasoning.

3. Given the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

(a) First read part (b) and then compute A^{-1}.

(b) Express A^{-1} as a product of elementary matrices.

(c) Use (a) to solve the matrix equation $A\vec{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.

4. (a) Complete the definition:

A set of vectors $\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_p}\}$ is <u>linearly independent</u> if

(b) Determine if the vectors $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \end{bmatrix} \right\}$. are linearly independent. Justify your answer.

(c) If A and B are both 3×3 matrices and the columns of B are linearly dependent vectors then show that the columns of AB are also linearly dependent. Explain and justify each step.

Hint: If $\vec{v_1}, \vec{v_2}, \vec{v_3}$ are the column vectors of B then we can write $AB = A\begin{bmatrix} \vec{v_1} & \vec{v_2} & \vec{v_3} \end{bmatrix} = \begin{bmatrix} A\vec{v_1} & A\vec{v_2} & A\vec{v_3} \end{bmatrix}$. Consider a linear combination $c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} = \vec{0}$.