

Sample Exam 1

Math 210, Fall 06

1. Consider the lines

$$\mathbf{r}_1(t) = (1 - t, 2 + t, t), \quad \mathbf{r}_2(s) = (s, 3 - s, 1 - 2s)$$

- (a) Find the intersection point of the two lines.
(b) Find the equation of the plane that contains both lines.
(c) Determine which line is perpendicular to the plane $-x + y + z = 0$.
2. (a) Sketch the graph of $f(x, y) = \sqrt{25 - x^2 - y^2}$ and the plane $z = 4$.
(b) Find a parametrization of the curve of intersection of the above surface and plane.
(c) Find the equation of the tangent line to this curve at $(0, 3, 4)$.
3. The temperature at a point (x, y) is

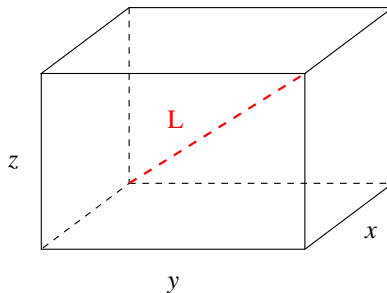
$$T(x, y) = \sin(xy).$$

An object is moving so that its position at time t is given by $x(t) = \sqrt{2}t$ and $y(t) = \pi t/4$. How fast is the temperature rising on the object's path when $t = 2$?

4. Find the following partial derivatives:

- (a) f_{rs} where $f(r, s, t) = rs \sin(e^t)$.
(b) $u_{yx}(1, 1)$ where $u(x, y) = x\sqrt{y + y^2}$.

5. If the length of the diagonal of a rectangular box must be L , what is the largest possible volume?



6. You are rowing on a lake whose depth is given by

$$d(x, y) = x e^{-x^2 - y^2},$$

where x and y are measured in meters and $(0, 0)$ is the center of the lake. If your boat is at $(1, 1)$ and is moving towards the center of the lake with the unit speed, is the lake getting shallower or deeper? At what rate?

7. **Extra Credit:** Find the maximum value of the function

$$f(x, y) = x^2 y$$

with the condition that $x^2 + y^2 = 1$.