Math 211Q test 1 **Name:**

50 pts

1. (10pts)Solve the following initial value problems

   (a) $\frac{dy}{dt} = -2ty + 4e^{-t^2} \qquad y(0) = 1$

   (b) $\frac{dy}{dt} = \frac{t^2}{y\sqrt{1+y^2}} \qquad y(0) = \sqrt{3}$

2. (5pts) Use Euler's method with a step size of $\frac{1}{4}$ to approximate $y(1)$ for the Initial value problem $\frac{dy}{dt} = y^2 + t^2 \qquad y(0) = 1.$

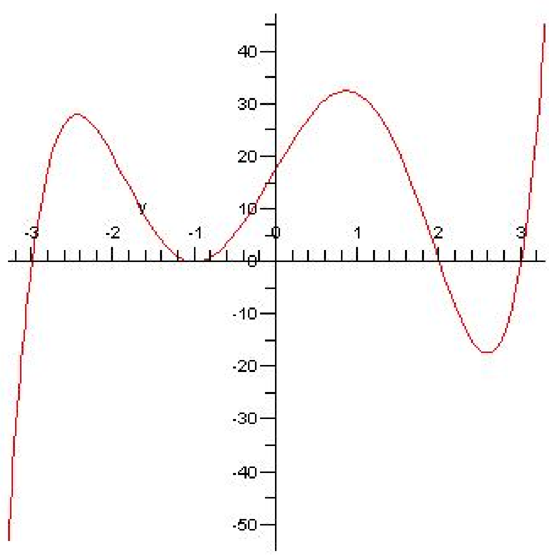
3. (10pts) For the differential equation $\dfrac{dy}{dt} = (y-3)^2 e^y \cos y$

(a) Find all equilibrium solutions on the interval $0 \le y \le 9$.

(b) Let y be the solution with initial condition $y(0) = 2$. Find $\lim_{t\to\infty} y(t)$.

(c) Let y be the solution with initial condition $y(0) = 4$. Find $\lim_{t\to-\infty} y(t)$.

4. (5pts) Below is a graph of $f(y)$ for the differential equation $\dfrac{dy}{dt} = f(y)$



Draw the phase line and label all the equilibrium points as source sink or node.

5. (10pts) The population of a certain species of fish in a certain pond is modeled by the differential equation

$$\frac{dP}{dt} = P(1 - \frac{P}{100}) - C$$

where P is the population of fish and C is the number of fish caught per year.

(a) Draw the phase lines for $C = 20$, and $C = 30$.

(b) How do you know there is a bifurcation point somewhere between 20 and 30?

(c) Find the bifurcation point.

(d) For an initial population of 100 fish, if 30 fish are caught per year what will happen to the fish population in the long run?

(e) For an initial population of 100 fish, if 20 fish are caught per year what will the population level out (give an exact number) at in the long run?

6. (10pts) A 85 gallon tank is filled with 25 gallons of pure water. Salt water containing 4 lbs per gallon is entering the tank at a rate of 5 gallons per minute. The well mixed tank is being emptied at a rate of 2 gallons per minute.

   (a) Construct an initial value problem to model the the amount of salt in the tank.

   (b) Solve the initial value problem.

   (c) How much salt is in the tank when the tank is full?

bonus (3pts) A differential equation $\frac{dy}{dt} = f(t, y)$, where $\frac{\partial f}{\partial y}$ and $f(t, y)$ are both continuous, has a solution $y(t) = t^3$. Does the differential equation have any equilibrium solutions? (explain.)