

Name: _____

Section: _____

Math 2410
Final Exam May 9 2014

You must show your work to receive credit.

1. Consider the autonomous differential equation

$$y' = -(y + 1)(y - 3)^3(y - 5)^3.$$

- (a) Compute the equilibrium solutions.
- (b) Sketch the phase line and classify the equilibria as sinks, sources, or nodes.
- (c) Describe the long term behavior of the solution to the above differential equation with initial condition $y(0) = 2$.

2. (a) Find the general solution of

$$y' + y = \sin(t)$$

(b) Solve the initial value problem

$$\begin{aligned}y' - (3/(2t + 1))y &= (2t + 1)^2 \\ y(0) &= 3.\end{aligned}$$

3. A cup of liquid is initially at a temperature of 90° in a room that is at 20° . After 10 minutes, it has cooled to 70° . Find the temperature at time t . When will it be 30° ?

4. (a) Consider the following system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= -x + 2y \\ \frac{dy}{dt} &= 3x - 2y .\end{aligned}$$

Find the general solution to the system.

- (b) Sketch the phase portrait for the system. Include a sketch of the solution with initial value $(0,2)$, that x vs t and y vs t .

5. (a) Consider the following system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= -x + 2y \\ \frac{dy}{dt} &= -2x - y.\end{aligned}$$

Find the general solution to the system.

(b) Sketch the phase portrait for the system.

6. Consider the equation

$$y'' - y = e^t$$

(a) Compute the general solution to the above equation.

(b) Solve for $y(0) = 0, y'(0) = 0$.

7. Find the solution for the spring-mass problem $y'' + 9y = \cos(3t)$. Solve with initial conditions $y(0) = 0, y'(0) = 0$. Use either Laplace transform or other method.

8. Consider the equation $y' + 4y = e^t - u_3(t)(t - 3)$ with initial conditions $y(0) = 1$.
Using the Laplace transform, find $y(t)$.

9. Consider the spring-mass system whose motion is governed by $y'' + 9y = f(t)$ with initial conditions $y(0) = 0, y'(0) = 0$ where the function $f(t)$ is given by $1 - u_2(t) \cos 3(t - 2)$.

Using the Laplace transform, find $y(t)$.

10. Consider the spring-mass system whose motion is governed by $y'' + 9y = \delta_5(t)$ with initial conditions $y(0) = 1, y'(0) = 0$. Find the position at time t .