Name:_____

Math 211
Final Exam - Fall 2006

You must show your work to receive credit.

1. A five gallon tank is full of water. We open a spigot so 1 gal. leaves the tank and introduce a mixture of 1/2 lb. per gal at 1 gal per minute. Assuming the mixture is well mixed, what's the concentration at time t?

2. Find the general solution of

$$y' + y = \sin(t)$$

Solve the initial value problem

$$\begin{aligned} y' - (3/(t+1))y &= (t+1)^2 \\ y(0) &= 3. \end{aligned}$$

3. The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\begin{aligned} \frac{dx}{dt} &= x(2 - x - y) \\ \frac{dy}{dt} &= y(6 - 2x - 2y). \end{aligned}$$

Draw the curves $x(t)$ and $y(t)$ if $x(0) = 3$ and $y(0) = 3$

4. Consider the linear system $\vec{Y}' = A\vec{Y}$ where $\vec{Y} = (x, y)$ and

$$A = \begin{pmatrix} -4 & -4 \\ -6 & -2 \end{pmatrix}$$

(a) Compute the eigenvalues of A.

(b) Classify the equilibrium at the origin (sink, spiral source, etc). Explain your answer.

(c) What is the general solution to the system? Sketch the phase plane.

5. Compute the general solution to the linear system $\vec{Y}' = A\vec{Y}$ where $\vec{Y} = (x, y)$ and

$$A = \begin{pmatrix} -1 & 4 \\ -4 & -1 \end{pmatrix}.$$

Sketch the phase plane.

6. Consider the spring-mass system whose motion is governed by

$$y'' + 6y' + 34y = 2e^{-t}.$$

(a) Compute the solution to the above equation if $y(0) = 0, y'(0) = 0$.

(b) Describe (in words) the long term behavior of the mass.

7. Find the general solution for the damped spring-mass problem $y'' + 4y = \sin(2t)$. Solve with initial conditions $y(0) = 0, y'(0) = 1$.

8. Consider the equation $y'' + 7y' + 6y = e^{-2t}$ with initial conditions $y(0) = 1, y'(0) = 0$. Using the Laplace transform, find $y(t)$.

9. Consider the equation $y'' + 8y' + 25y = e^{-t}$ with initial conditions $y(0) = 1, y'(0) = 0$. Using the Laplace transform, find $y(t)$.

10. Consider the spring-mass system whose motion is governed by $y'' + 9y = 1 + u_3(t)\sin(t-3)$ with initial conditions $y(0) = 0, y'(0) = 0$. Using the Laplace transform, find $y(t)$.