

Name: _____

Math 211
Exam 2

You must show your work to receive credit.

1. Consider the linear system $\vec{Y}' = A\vec{Y}$ where

$$A = \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix}$$

(a) Compute the eigenvalues of A and find the general solution. Sketch the phase plane.

(b) find the solution for the initial value problem $Y_1(0) = 1, Y_2(0) = 0$.

2. Consider the linear system $\vec{Y}' = A\vec{Y}$ where

$$A = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix}$$

(a) Compute the eigenvalues of A and find the general solution.

(b) Sketch the phase plane.

(c) find the solution for the initial value problem $Y_1(0) = 1, Y_2(0) = 0$.

3. Find the solution for the problem $y'' + 4y' + 5y = 0, y(0) = 1, y'(0) = 2$

4. The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\begin{aligned}\frac{dx}{dt} &= x(2 - x - y) \\ \frac{dy}{dt} &= y(3 - x - 3y).\end{aligned}$$

Draw the curves $x(t)$ and $y(t)$ if $x(0) = 1$ and $y(0) = 1$