

Name: _____

Math 2410
Exam 1

You must show your work to receive credit. Each problem for 20 points.

1. Consider the autonomous differential equation

$$y' = y^2(y - 3)(y - 5)^3.$$

- (a) Compute the equilibrium solutions.
- (b) Sketch the phase line and classify the equilibria as stable (sink), unstable (source), or nodes.
- (c) Describe the long term behavior of the solution to the above differential equation with initial condition $y(0) = 4$.

2. Find the general solution of

$$y' + 2y = t + 1$$

Solve the initial value problem

$$\begin{aligned}y' - (3/(4t + 1))y &= 4t + 1 \\ y(0) &= 3.\end{aligned}$$

3. For the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= (-3x - y) \\ \frac{dy}{dt} &= (-2x - 2y).\end{aligned}$$

- (a) Find the general solution.
- (b) Find the solution if $x(0) = 1$ and $y(0) = 2$.

4. The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\begin{aligned}\frac{dx}{dt} &= x(6 - x - y) \\ \frac{dy}{dt} &= y(4 - x - 2y).\end{aligned}$$

Draw the curves $x(t)$ vs. t and $y(t)$ vs. t if $x(0) = 5$ and $y(0) = 1$

5. Consider the differential equation

$$y' = ty^{\frac{2}{5}}.$$

(a) Compute the solution to the above differential equation.

(b) Is there a *unique* solution $y(t)$ to the above differential equation such that $y(0) = 0$? *Why or why not?*

(c) Is there a *unique* solution $y(t)$ to the above differential equation such that $y(0) = 1$? *Why or why not?*

6. A cup of liquid is initially at a temperature of 90° in a room that is at 20° . After 10 minutes, it has cooled to 80° . Find the temperature at time t . When will it be 30° ?

7. For the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= (-2x - y) \\ \frac{dy}{dt} &= -3y.\end{aligned}$$

Find the general solution.