

Name: _____

Math 211
Exam 1

You must show your work to receive credit.

1. Consider the autonomous differential equation

$$y' = y(y - 3)^2(y - 5).$$

- (a) Compute the equilibrium solutions.

- (b) Sketch the phase line and classify the equilibria as sinks, sources, or nodes.

- (c) Describe the long term behavior of the solution to the above differential equation with initial condition $y(0) = 4$. Draw the graph of y vs. t .

2. Find the general solution of

$$y' + 3y = e^t$$

Solve the initial value problem

$$\begin{aligned}y' - (2/(2t + 1))y &= 2 \\ y(0) &= 3.\end{aligned}$$

3. The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\begin{aligned}\frac{dx}{dt} &= x(2 - x - y) \\ \frac{dy}{dt} &= y(4 - x - 4y).\end{aligned}$$

Draw the curves $x(t)$ and $y(t)$ if $x(0) = 1$ and $y(0) = 1$

4. Consider the differential equation

$$y' = y^{\frac{1}{3}} t^{\frac{2}{5}}.$$

(a) Compute the solution to the above differential equation.

(b) Is there a *unique* solution $y(t)$ to the above differential equation such that $y(0) = 0$? *Why or why not?*

(c) Is there a *unique* solution $y(t)$ to the above differential equation such that $y(0) = 1$? *Why or why not?*

5. a ten gallon tank is full of pure water. We open a spigot so one gal. leaves the tank and introduce a mixture of $1/2$ lb. per gal at 1 gal per minute. Assuming the mixture is well mixed, what's the concentration at time t ?