

Name: _____

Math 211
Exam 1

You must show your work to receive credit.

1. Consider the autonomous differential equation

$$y' = y(y - 3)^2(y - 5).$$

(a) Compute the equilibrium solutions.

(b) Sketch the phase line and classify the equilibria as sinks, sources, or nodes.

(c) Describe the long term behavior of the solution to the above differential equation with initial condition $y(0) = 4$.

2. Find the general solution of

$$y' + 2y = t^2 + 1$$

Solve the initial value problem

$$\begin{aligned}y' + (2/t)y &= t^2 \\ y(0) &= 3.\end{aligned}$$

3. Identify the four direction fields with four of the eight systems given below.

4. The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

$$\begin{aligned}\frac{dx}{dt} &= x(6 - x - y) \\ \frac{dy}{dt} &= y(4 - x - 2y).\end{aligned}$$

Draw the curves $x(t)$ and $y(t)$ if $x(0) = 1$ and $y(0) = 1$

5. Consider the differential equation

$$y' = yt^{\frac{2}{5}}.$$

(a) Compute the solution to the above differential equation.

(b) Is there a *unique* solution $y(t)$ to the above differential equation such that $y(0) = 0$? *Why or why not?*

(c) Is there a *unique* solution $y(t)$ to the above differential equation such that $y(0) = 1$? *Why or why not?*

6. Find the general solution to the system

$$\begin{aligned}\frac{dx}{dt} &= 2x + y \\ \frac{dy}{dt} &= y.\end{aligned}$$