You must show your work to receive credit.

1. Consider the autonomous differential equation

\[ y' = y(y - 3)^2(y - 5). \]

(a) Compute the equilibrium solutions.

(b) Sketch the phase line and classify the equilibria as sinks, sources, or nodes.

(c) Describe the long term behavior of the solution to the above differential equation with initial condition \( y(0) = 4 \). Draw the graph of \( y \) vs. \( t \).
2. Find the general solution of

\[ y' + 3y = e^t \]

Solve the initial value problem

\[ y' - \frac{2}{2t + 1}y = 2 \]
\[ y(0) = 3. \]
3. The following system describe a pair of competing species. Describe the long-time likely outcome of the competition by plotting the direction field.

\[
\frac{dx}{dt} = x(2 - x - y) \\
\frac{dy}{dt} = y(4 - x - 4y).
\]

Draw the curves \( x(t) \) and \( y(t) \) if \( x(0) = 1 \) and \( y(0) = 1 \).
4. Consider the differential equation

\[ y' = y^3 t^2. \]

(a) Compute the solution to the above differential equation.

(b) Is there a unique solution \( y(t) \) to the above differential equation such that \( y(0) = 0 \)? Why or why not?

(c) Is there a unique solution \( y(t) \) to the above differential equation such that \( y(0) = 1 \)? Why or why not?
5. A ten gallon tank is full of pure water. We open a spigot so one gal. leaves the tank and introduce a mixture of 1/2 lb. per gal at 1 gal per minute. Assuming the mixture is well mixed, what’s the concentration at time $t$?