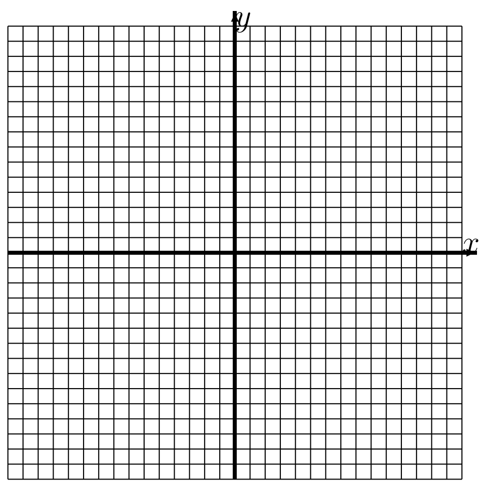
Final Exam

Instructions: You may choose any 3 of the 6 problems to complete (the bonus does not count as a problem)

Special Note: Time is a relative thing. Relative to the present (at choose to least some of us hope/believe that) there cannot be negative time, however when modeling something that has happened in the past, it is fair to use a different reference point, and so you can have negative time.

1. (Civil) It is standard in some construction to have a beam hinged to a wall at its left end and simply supported at its right end. A force is called an axial load applied at the right end. If enough force is given then the beam will buckle, this being called its buckling load. A senior engineer is designing a project in which such a component is being used. If he has computed the that the buckling load for a specific beam is given as the smallest positive solution to $(1/6)x^3 + (3/4) = (1/120)x^5 + x$ then find the buckling load to within 6 significant digits.

2. (Electrical) A valid model for AC circuits is by a simple sinusoidal (sine) wave, however in practice most waves are not so structured. Consider a waveform where the voltage given at time x is $V(x) = \frac{-1}{24}x^4 + \frac{3}{2}x^2 + \frac{3}{4}$ over the time interval $(-5.5, 5.5)$. First graph the waveform on the graph below. Next find the maximal and minimal voltage of this circuit. Finally compute a measure of the strength of the circuit called the "peak to peak" voltage, this being the difference between consecutive maximal peak and a minimal peak.



3. (CSE) Write using pseduo-code (or whatever language you prefer), a algorithm for computing the zero's of a given polynomial using a given an initial guess to within 10 significant digits. Then Trace the program, (give the memory values for) for a run of $f(x) = x^{11} - 2x^2 + 1$ and an initial guess of $x = -1$.

4. (ME) A hybrid fuel-electric powered autonomous agent is sent to Io (one of the moon's of Jupiter). It will run off a power source charged using both fuel and electric battery. Because fuel is expensive and dangerous to transport we would like to have the module have twice as much of its charge coming from the electrical battery as the fuel cell, (that is the total charge we should use charge from the battery twice every time we charge from the fuel cell). If the power donated from the battery at time x is $B(x) = x^5 - \frac{3}{2}x^2 + 45$ and the power donated by the fuel cell is $F(x) = x^6 + \frac{13}{2}x^4 - x^3 + 2x - 1$. The unit cannot make its measurements for the day until we have our power supply charged to at least 100. Find the time that this can happen to within $\frac{1}{1000}sec$. If we charge too much we may lose our power supply for the mission and so we will need to turn off the charge before it reachers a charge of 150. When will this happen to within $\frac{1}{1000}sec$? (Hint: Use a large ymin and ymax for window size, something like 100)

5. (BME) A viral infection is being diagnosed and you are the engineer assigned to help setup experiments to test drugs as possible treatments. Your senior engineer chart the rate of the virus reproduction within a body under the condition of two drugs as $f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x + 3$ and $g(x) = \frac{1}{3}x^3 - \frac{9}{2}x^2 + 14x + 1$. Under the time (in hours) window $(0, 10)$ which drug was the most limiting of the viral reproduction?

6. (Chem E) Placing chemicals together do not simply get them to react, one often needs to add energy to spark the reaction. The energy required is called the activation energy of the reaction and is given in KJ/mol This can be computed by the maximum of the of energy as a function over the path of the reaction. Let's say experimentally a chemical engineer computes this function to be approximately $f(x) = \frac{2}{3}x^3 + \frac{3}{2}x^2 - 5x + 4$. Find the activation energy where the time window is $(-4, 1.5)$. Let's say that this reaction is then coupled with another reaction, both using the same energy source. What is the greatest amount of energy available at any given point in time to the second reaction?

Bonus: (Math)Given that a function f is differentiable at a point $x = a$, show that f is also continuous at $x = a$. Is the converse true, that is given a function f that is continuous at $x = a$ then must it also be differentiable at $x = a$? You can either show this if it is true (and justify your work), or you if it is false, give and example of a function which is continuous at $x = a$ and not differentiable at $x = a$.