

Mathematics 114Q
Practice Exam 2, Spring 2005

Name: _____

Answer the questions in the space provided. If you need more space, continue on the back of the page. All answers must include supporting work or an explanation of your reasoning. Without supporting work, even correct answers may be marked wrong. Use of a calculator is not allowed. This practice exam has ?? questions, for a total of ?? points.

1. (45 points) Calculate the following integrals.

(a) $\int x \cos(x^2) dx$

(b) $\int x^2 \cos(x) dx$

(c) $\int \cos^2(x) dx$

$$(d) \int \frac{x+2}{x^2+x} dx$$

$$(e) \int \frac{1}{x^3-4x} dx$$

$$(f) \int \frac{1}{\sqrt{9-x^2}} dx$$

$$(g) \int \frac{2x^2 - x - 1}{(x^2 + 1)(x - 2)} dx$$

$$(h) \int \frac{1}{x^2 + 2x + 2} dx$$

$$(i) \int \frac{3}{\sqrt{2x - x^2}} dx$$

2. (30 points) Determine whether or not the following improper integrals converge. Evaluate, if possible.

(a) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(b) $\int_0^1 \frac{1}{\sqrt{x^4 + x}} dx$

(c) $\int_1^\infty \frac{1}{\sqrt{x^4 + x}} dx$

$$(d) \int_e^\infty \frac{1}{(\ln x)^2} \cdot \frac{1}{x} dx$$

$$(e) \int_0^\infty e^{-x} dx$$

$$(f) \int_0^\infty e^{-x^2} dx$$

3. (15 points) Find the exact area under the following curves:

(a) $y = \frac{1}{x^{3/2}}$ for $x \geq 1$.

(b) $y = xe^{-2x}$ for all $x \geq 0$.

(c) The graph of y , where $y = \frac{1}{\sqrt{x}}$ for $x \leq 1$ and $y = \frac{1}{x^2}$ for $x > 1$.

4. (15 points) Find the area of the following regions:

(a) The region bounded by $y = x$ and $y = \sqrt{x}$.

(b) The region bounded by $y = \sqrt{x}$ and $y = x^2$.

(c) The region bounded by $x = 0$, $y = 1 - x$ and $y = x^2 - 1$.

5. (10 points) Find the following arc lengths:

(a) The curve $f(x) = \frac{2}{3}\sqrt{x^3}$ from $x = 0$ to $x = 1$.

(b) The parametric curve $x = \sin(e^t)$, $y = \cos(e^t)$, from $t = 0$ to $t = 1$.

6. (5 points) Find the volume of the solid region obtained by rotating the region bounded by $y = x^2 + 1$, $y = 0$, and $x = 1$ around the x -axis.

7. (5 points) Find the volume of the solid region obtained by rotating the region bounded by $y = 1 - x^2$, $y = 0$, and $x = 1$ around the x -axis.

8. (5 points) Find the mass of a rod of length 5 cm with density $\delta(x) = 1 + x^2$ gm/cm at a distance of x cm from the left end.

9. (5 points) Find the mass of a rod of length 10 cm with density $\delta(x) = \frac{1}{1+x}$ gm/cm at a distance of x cm from the left end.

10. (5 points) A water tank has dimensions (in meters) 10 long, 15 high, and 20 wide. If the tank is full, how much work does it take to pump all the water out?

11. (5 points) A water tank has dimensions (in meters) 20 long, 10 high, and 15 wide. If the tank is full, how much work does it take to pump all the water out?

12. (5 points) A water tank has dimensions (in meters) 10 long, 20 high, and 15 wide. If the tank is full, how much work does it take to pump all the water out?