1 Sample Gateway I

1. $\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \sin\theta\cos\theta d\theta$.

   First we find $\int \sin\theta\cos\theta d\theta$. We do this using the substitution $u = \sin\theta$, Then $du = \cos\theta d\theta$ and we have that $\int \sin\theta\cos\theta d\theta = \int u du = \frac{1}{2}u^2 = \frac{1}{2}\sin^2\theta$. So now we evaluate $\frac{1}{2}[sin^2\theta]_{\theta=\frac{\pi}{2}}^{\theta=\frac{2\pi}{3}} = \frac{1}{2}[\sin(\frac{2\pi}{3})^2 - \sin(\frac{\pi}{2})^2] = \frac{1}{2}[(\frac{\sqrt{3}}{2})^2 - 1^2] = \frac{1}{2}[\frac{3}{4} - 1] = \frac{1}{2} * \frac{1}{4} = \frac{1}{8}$

2. $\int_0^4 \sqrt{x}(x^2 - 4)dx$

   Again first we find $\int \sqrt{x}(x^2 - 4)dx$. We do this by the power rule. So we compute $\int x^{\frac{5}{2}} - 4x^{\frac{1}{2}}dx = \frac{2}{7}x^{\frac{7}{2}} - 4 * \frac{2}{3}x^{\frac{3}{2}}$. Now evaluating we obtain $\frac{2}{7}(4)^{\frac{7}{2}} - 4 * \frac{2}{3}4^{\frac{3}{2}} = \frac{2^8}{7} - \frac{8^2}{3} = \frac{3*256-7*64}{21} = \frac{320}{21} = 15$.

3. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

   We use substitution $u = \sqrt{x}$ which gives $du = \frac{1}{2} * \frac{1}{\sqrt{x}}$, thus we have $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int e^u du = 2e^u + C = 2e^{\sqrt{x}}$.

4. $\int e^{\frac{1}{2}ln(x^2)}dx$

   First we use some algebraic simplification and find that $e^{\frac{1}{2}ln(x^2)} = e^{ln(x)} = e^x$, so $\int e^{\frac{1}{2}ln(x^2)}dx = \int e^x dx = e^x + C$.

5. $\int \frac{x^4}{x+1}dx$

   Here we first divide to make this proper and find that $(x+1) * (x^3 - x^2 + x - 1 + \frac{1}{x+1}) = x^4$. Thus $\int \frac{x^4}{x+1}dx = \int(x^3 - x^2 + x - 1 + \frac{1}{x+1})dx = \frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 - x + ln|x+1| + C$.

6. $\int x\sqrt{(x^2+16)}dx$

   Here use a substituition $u = x^2 + 16$ which gives $du = 2xdx$ or that $xdx = \frac{1}{2}du$. Now doing the substitution we have $\int x\sqrt{(x^2+16)}dx = \frac{1}{2}\int \sqrt{u}du = \frac{1}{2} * \frac{2}{3}u^{\frac{3}{2}} = \frac{1}{3}(x^2+16)^{\frac{3}{2}}$.

7. $\int_0^{\frac{\pi}{4}} \tan^2(x)dx$

   Here it is helpful to recall that $\tan^2(x) + 1 = \sec^2(x)$ so that we can rewrite our integral $\int \tan^2(x)dx = \int(\sec^2(x) - 1)dx = \tan(x) - x$. Now evaluating we find our answer $\tan(\frac{\pi}{4}) - \frac{\pi}{4} - \tan(0) + 0 = 1 - \frac{\pi}{4}$.

2 Sample Gateway II

1. $\int_0^{\frac{\pi}{3}} \sin(\theta)d\theta$ First we find that $\int \sin(\theta)d\theta = -\cos(\theta)$, This gives that our definite integral is $-\cos(\frac{\pi}{3}) + \cos(0) = -\frac{1}{2} + 1 = \frac{1}{2}$.

2. $\int \frac{ln(2x)}{x}dx$

   We find the substitution $u = ln(2x)$ to be helpful in that $du = \frac{2dx}{2x} = \frac{dx}{x}$. Therefore we have that $\int \frac{ln(2x)}{x}dx = \int udu = \frac{1}{2}u^2 + C = \frac{1}{2}ln(2x)^2 + C$.

3. $\int_0^1 (\frac{2}{x+2} - \frac{x+2}{2})dx$

   First we evaluate $\int(\frac{2}{x+2} - \frac{x+2}{2})dx$ Splitting this up and using the power rule and the substitution $u = x + 2$ we find that $\int(\frac{2}{x+2} - \frac{x+2}{2})dx = 2ln|x+2| - \frac{1}{2}(\frac{1}{2}x^2 + 2x) = 2ln|x+2| - \frac{1}{4}x^2 - 2$. Now we evaluate the definite part and conclude with the answer $2ln|3| - \frac{1}{4} - 2 - 2ln|2| + 2 = 2ln|\frac{3}{2}| - \frac{1}{4}$.

4. $\int xe^{x-1}dx$

   We find that this integral is easiest done by parts, with the substitutions $u = x$ which gives $du = dx$, and $dv = e^{x-1}dx$ which gives $v = e^{x-1}$. Using the by parts formula we find that $\int xe^{x-1}dx = xe^{x-1} - \int e^{x-1}dx = xe^{x-1} - e^{x-1} + C$.

5. $\int \frac{x^3+1}{\sqrt{x}}dx$

   This integral will be made easy by using some algebra and writing $\frac{x^3+1}{\sqrt{x}} = x^{\frac{5}{2}} + x^{\frac{-1}{2}}$ which gives $\int \frac{x^3+1}{\sqrt{x}}dx = \int x^{\frac{5}{2}} + x^{\frac{-1}{2}} = \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{1}{2}} + C$.

6. $\int x\cos(x^2+1)dx$

   Using the substitution $u = x^2 + 1$ we find that $du = 2xdx$ which gives $\int x\cos(x^2+1)dx = \frac{1}{2}\int \cos(u)du = \frac{1}{2}\sin(u) + C = \frac{1}{2}\sin(x^2+1) + C$.

7. $\int_0^{\frac{\pi}{2}} \tan(2x)dx$.

   First we find $\int \tan(2x)dx$ we find this by writing $\tan(2x) = \frac{\sin(2x)}{\cos(2x)}$. We find the substitution $u = \cos(2x)$ gives $du = 2\sin(2x)dx$ and we find that $\int \tan(2x) = \int \frac{\sin(2x)}{\cos(2x)} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}ln|u| = \frac{1}{2}ln|\cos(2x)|$. Now evaluating we find that $\frac{1}{2}(ln|\cos(2 * \frac{\pi}{2})| - ln|\cos(0)|) = \frac{1}{2}(ln|-1| - ln|1|) = 0$.