

112 Practice Test 3

You may use calculators, there will be partial credit for work shown. Take under timed conditions (1 hr)

[10] 1. Let $f(x) = \ln(\sin(\pi x + \frac{\pi}{4}))$

a). Find average rate of change between $a = 0$, and $b = \frac{1}{2}$
 $f(\frac{1}{2}) = \ln(\frac{\sqrt{2}}{2}) = f(0)$ so Avg rate of change = $\frac{f(b)-f(a)}{b-a} = 0$.

b). Find instantaneous rate of change at $a = \frac{1}{2}$
 $f'(x) = \pi * \frac{\cos(\frac{3\pi}{4})}{\sin(\frac{3\pi}{4})} = -\pi$

c). Find line tangent to f at $(\frac{1}{2}, \ln(\frac{\sqrt{2}}{2}))$
 $(y - \ln(\frac{\sqrt{2}}{2})) = -\pi(x - \frac{1}{2})$

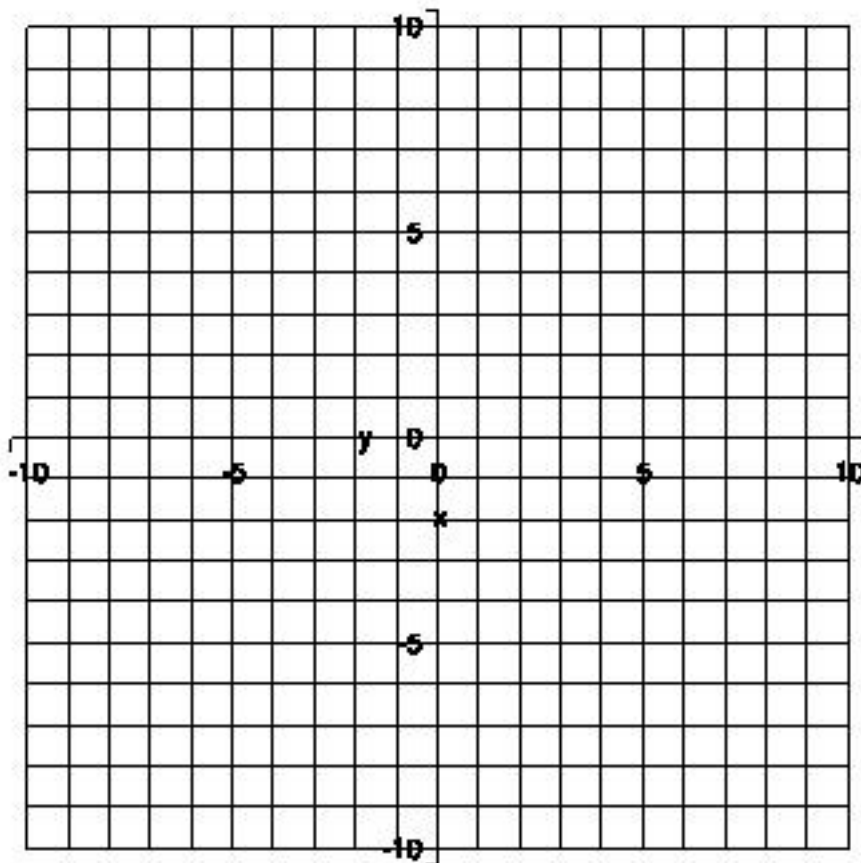
[10] 1. Let $f(x) = e^{x^2} - e^{-x^2}$

a). Find average rate of change between $a = 0$, and $b = 1$
 $f(1) = e - \frac{1}{e}$, $f(0) = 0$ so Avg rate of change = $\frac{f(b)-f(a)}{b-a} = e - \frac{1}{e}$

b). Find instantaneous rate of change at $a = 0$
 $f'(x) = 2xe^{x^2} + 2xe^{-x^2}$ so $f'(0) = 0$

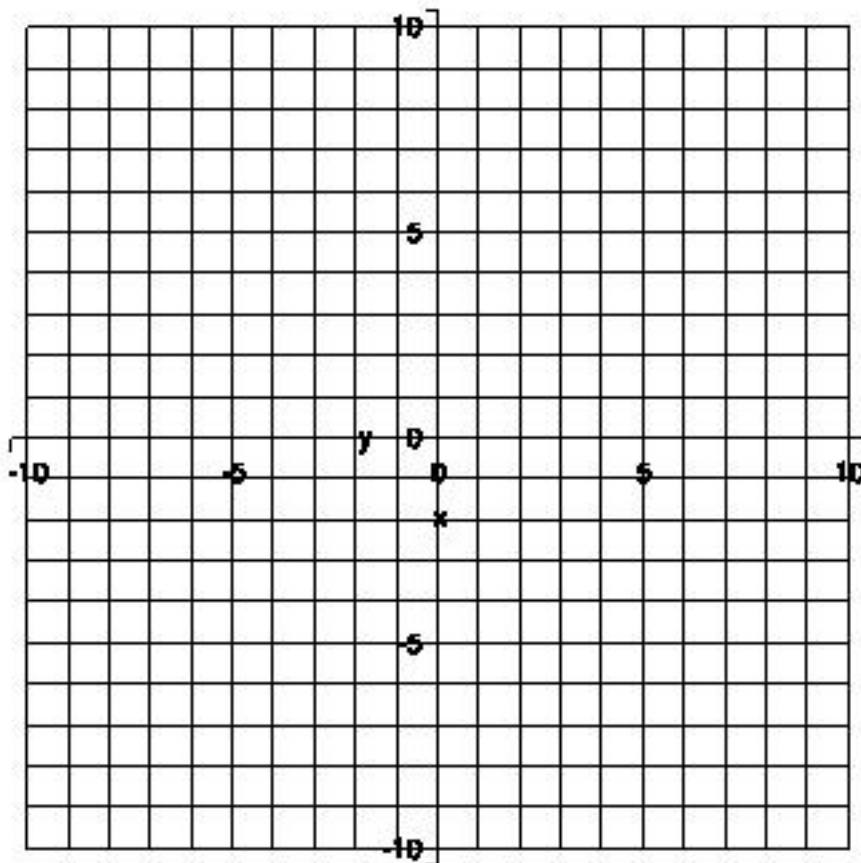
c). Find line tangent to f at $(0,0)$
Slope 0, so line is horizontal going through $(0,0)$ must be x-axis

[5] 2. For $f(x)$ given below



- a). Graph $f'(x)$
- b). List (in set notation) points where f is not differentiable
 $\{-8, -5, 4\}$
- c). List (in set notation) points where f is not continuous
 $\{-5\}$
- d). List (in set notation) points where f' is not continuous
 $\{-8, -5, 4\}$

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[10] 3. The height of a falling body is given by $h(t) = -32t^2 + 360t + 1000$, find the height when the object starts to fall
 $h'(t) = -64t + 360$, so when $h'(t) = 0 \Rightarrow t = 5$ and $h(5) = 2640$

[10] 3. Let $f(x) = \frac{1}{3}x^3 - 10x^2 + 30x - 100$, find all local extrema
 $h'(t) = x^2 - 20x + 30$ which has no real roots.

[10] 4. Consider the following table:

x	f	g	h	f'	g'	h'
-1	Π	0	3	$\frac{1}{2}$	1	2
0	-1	-5	$\frac{\Pi}{4}$	$\frac{1}{4}$	17	$2e$
1	$\sqrt{2}$	-6	9	$\frac{1}{8}$	4Π	122

a). Find $\frac{d}{dx}(f(x) + g(x))$ at $x = 1$
 $4\pi + \frac{1}{8}$

b). Find $\frac{d}{dx}(f(g(x)))$ at $x = -1$
 0

c). Find $\frac{d}{dx}(\cos(f(x)))$ at $x = -1$
 0 remember $\sin(\pi) = 0$

d). Find $\frac{d}{dx}(e^h(x))$ at $x = 0$
 $2e * e^{\frac{\pi}{4}}$

e). Find $\frac{d}{dx}(h(g(x)))$ at $x = 1$
 $2\pi * h'(-6)$

f). Find $\frac{d}{dx}(f(x) - h(x)g(x))$ at $x = 0$
 $\frac{1}{4} + 10e - \frac{17\pi}{4}$

g). Find $\frac{d}{dx}(\ln((f(x))^2))$ at $x = 1$
 $\frac{\sqrt{2}}{8}$

[10] 5. Let $f(x) = \frac{1}{12}x^4 + \frac{1}{6}x^3 - 3x^2 + 17x - 122q$ find intervals for when $f(x)$ is concave down.

$f''(x) = x^2 + x - 6 = 0$ and we have roots $x = -3, 2$, finding the sign of $f''(x)$ we have that $f''(x) < 0$ on $(-3, 2)$

[10] 5. Let $f(x) = \frac{1}{16}x^4 - \frac{1}{9}x^3 + 10x^2 + a\sqrt{\pi}x - 122a^2\pi$ find intervals for when $f(x)$ is concave up.

Not real roots

[5] 7. Let $y = f(x)$, find $\frac{dy}{dx}$ if $y^3 \sqrt[3]{x^2} - e^{y^2} = xy$

$$\frac{dy}{dx} = \frac{\frac{2}{3}y^3x^{-\frac{1}{3}} - y}{x + 2ye^{y^2} - 2y^2x^{\frac{2}{3}}}$$

[5] 7. Let $y = f(x)$, find $\frac{dy}{dx}$ if $\frac{x}{y} - y^2 = \ln y$

$$\frac{dy}{dx} = \frac{y^{-1}}{xy^{-1} + 2y + \frac{1}{y}}$$

[5] 7. Let $y = f(x)$, find $\frac{dy}{dx}$ if $(x + y)^2 + \sin(y) = (xy)^3$

$$\frac{dy}{dx} = \frac{3x^2y^3 - 2(x+y)}{2(x+y) + \cos(y) - 3x^3y^2}$$

[5] 7. Let $y = f(x)$, find $\frac{dy}{dx}$ if $\tan(y)\cos(x) + e^{xy} = \tan(xy^2)$

$$\frac{dy}{dx} = \frac{y^2 \sec^2(xy^2) + \sin(x)\tan(y) - ye^{xy}}{\cos(x)\sec^2(y) - 2xy\sec^2(xy^2) + xe^{xy}}$$