

MASSIVE MATH 112 GATEWAY REVIEW

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ABSTRACT. This document is meant as a review of the concepts needed to pass the gateway exam for Math 112 at the University of Connecticut. The concepts are laid out similar to how the concepts have been laid out in the past. We have composed this document as a resource to students whom are having trouble with these concepts, and to allow the students a resource of practice problems with solutions, along with a admittedly modest description of how to solve such problems.

1. LINEAR EQUATIONS

The general form for a linear equations is

$$y = mx+b$$

where m is interpreted as slope, and b is the number such that the line intersects the y -axis at the point $(0,b)$. If a line passes through two points, say (x_1,y_1) and (x_2,y_2) , then the slope of the that line will be given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Once we have found the slope of the line we are trying to find, then we can find the intercept by plugging in one of the points and solving for the intercept. One may also solve the problem of finding the line through a point (x_1,y_1) given a slope m by using the point slope formula:

$$(y - y_1) = m(x - x_1)$$

If $y=mx+b$ is any line, then any line parallel to it will have the same slope, i.e., slope m , and any line perpendicular to it will have a slope of negative reciprocal, i.e., slope $\frac{-1}{m}$.

1.1. Example 1. Problem: Find the line between points $(1,1)$ and $(-3,2)$.

Solution: We first find the slope $m = \frac{2-1}{-3-1} = \frac{-1}{4}$. Then we plug in one of the points to obtain the intercept, $1 = \frac{-1}{4} + b \Rightarrow b = \frac{5}{4}$ so our equation is: $y = \frac{-1}{4} + \frac{5}{4}$.

1.2. Example 2. Problem: Find the lines parallel and perpendicular to $3y - 5 = \frac{2x+1}{3}$ through the point (3,2)

Solution: We begin by finding the slope of the line given, so working it into standard form we obtain $y = \frac{2}{9}x + \frac{16}{9}$.

For the parallel line we can use the same slope, so we have to find the line with slope $\frac{2}{9}$ and through the point (3,2). So we plug in to find the intercept $2 = 3\frac{2}{9} + b \Rightarrow 2 = \frac{2}{3} + b \Rightarrow b = \frac{4}{3}$ so we have the equation $y = \frac{2}{9} + \frac{4}{3}$

For the perpendicular line we use the negative reciprocal of the slope, or that we use the slope of $m = \frac{-9}{2}$ so that now we plug in our point to find the intercept $2 = 3\frac{-9}{2} + b \Rightarrow b = 2 + \frac{27}{2} \Rightarrow b = \frac{31}{2}$

1.3. Practice Problems.

- 1) Find the slope and y intercept of $\frac{y+2}{x-3} = \frac{1}{2}$
- 2) Find the line parallel to $\frac{y+2}{x-3} = \frac{1}{2}$ going through (1,2)
- 3) Find the line perpendicular to $\frac{y+2}{x-3} = \frac{1}{2}$ going through (1,2)
- 4) Find the slope and y intercept of $x + 2y = -1$
- 5) Find the line parallel to $x + 2y = -1$ through (-3,2)
- 6) Find the line perpendicular to $x + 2y = -1$ through (-3,2)
- 7) Find the slope and y intercept of $2x + 5y = 1$
- 8) Find the line parallel to $2x + 5y = 1$ through (5,1)
- 9) Find the line perpendicular to $2x + 5y = 1$ through (2,5)
- 10) Find the slope and y intercept of $\frac{x-5}{5} = \frac{y-2}{2}$
- 11) Find the line parallel to $\frac{x-5}{5} = \frac{y-2}{2}$ through (5,1)
- 12) Find the line perpendicular to $\frac{x-5}{5} = \frac{y-2}{2}$ through (2,1)

- 13) Find the slope and y intercept of $x - (3 - 2y) = -4$
- 14) Find the line parallel to $x - (3 - 2y) = -4$ through $(-3,2)$
- 15) Find the line perpendicular to $x - (3 - 2y) = -4$ through $(-3,2)$
- 16) Find the slope and y intercept of $2y - (4 - 3x) = 12$
- 17) Find the line parallel to $2y - (4 - 3x) = 12$ through $(2,1)$
- 18) Find the line perpendicular to $2y - (4 - 3x) = 12$ through $(3,1)$
- 19) Find the slope and y intercept of $\frac{x+6}{6} = \frac{2y+8}{2}$
- 20) Find the line parallel to $\frac{x+6}{6} = \frac{2y+8}{2}$ through $(6,3)$
- 21) Find the line perpendicular to $\frac{x+6}{6} = \frac{2y+8}{2}$ through $(6,3)$
- 22) Find the slope and y intercept of $6y - (1 + 4x) = 13$
- 23) Find the line parallel to $6y - (1 + 4x) = 13$ through $(3,2)$
- 24) Find the line perpendicular to $6y - (1 + 4x) = 13$ through $(2,3)$
- 25) Find the slope and y intercept of $x - y + 6 = x + 3y$
- 26) Find the line parallel to $x - y + 6 = x + 3y$ through $(1,2)$
- 27) Find the line perpendicular to $x - y + 6 = x + 3y$ through $(1,2)$
- 28) Find the equation of the line passing through $(1,2)$ and $(5,3)$
- 29) Find the line parallel to $6x + 4y = 1$ through $(3,3)$
- 30) Find the line perpendicular to $6x + 4y = 1$ through $(3,3)$
- 31) Find the equation of the line through the points $(0,-1)$ and $(2,5)$
- 32) Find the line parallel to $y = 7 - 3(2x - y)$ through $(1,1)$

- 33) Find the line perpendicular to $y = 7 - 3(2x - y)$ through (1,1)
- 34) Find the equation of the line through the points $(\frac{1}{3}, \frac{2}{3})$ and $(\frac{3}{2}, \frac{5}{3})$
- 35) Find the line parallel to $3y - (2 - 3x) = 13$ through (1,1)
- 36) Find the line perpendicular to $3y - (2 - 3x) = 13$ through (1,1)
- 37) Find the equation of the line through the points (1,1) and (2,9)
- 38) Find the line parallel to $\frac{x}{4} = \frac{y}{2}$ through (2,4)
- 39) Find the line perpendicular to $\frac{x}{4} = \frac{y}{2}$ through (2,4)
- 40) Find the equation of the line through the points (0,4) and (2,8)
- 41) Find the line parallel to $y = 2$ through (0,0)
- 42) Find the line perpendicular to $y = 2$ through (0,0)
- 43) Find the equation of the line through the points (-1,-2) and (1,2)
- 44) Find the line parallel to $\frac{2x+3}{1-y} = 3$ through (0,0)
- 45) Find the line perpendicular to $\frac{2x+3}{1-y} = 3$ through (0,0)
- 46) Find the equation of the line through the points (0,8) and (8,0)
- 47) Find the line parallel to $5x + 3y = 8$ through (1,1)
- 48) Find the line perpendicular to $5x + 3y = 8$ through (1,1)
- 49) Find the slope and intercept of $y=3$
- 50) Find the line perpendicular to $y=3x$ through (1,1)

2. COMPOSITION OF FUNCTIONS

Composition is a process by which we take two functions and creating another function. So say we start with two functions, $f(x)$ and $g(x)$. Then the composition $(f \circ g)(x)$ is obtained by 'evaluating' $f(x)$ at $g(x)$ or finding $f(g(x))$. Here order is important as $(f \circ g)(x)$ is not always $(g \circ f)(x)$.

Similar problems that appear on the gateway is when g is given implicitly instead of explicitly (see example 2).

2.1. Example 1. Problem: Let $f(x) = 2\sqrt{x+1}$ and $g(x) = x+2$ find $(f \circ g)(x)$ and $(g \circ f)(x)$

Solution: $(f \circ g)(x) = f(g(x)) = f(x+2) = 2\sqrt{(x+2)+1} = 2\sqrt{(x+3)}$ $(g \circ f)(x) = g(f(x)) = g(2\sqrt{(x+1)}) = 2\sqrt{(x+1)} + 2$

2.2. Example 2. Problem: Let $f(x) = x^2 + 5$ find $\frac{f(x+h)-f(x)}{h}$

Solution: $f(x+h) = (x+h)^2 + 5$ so $\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 + 5 - (x^2 + 5)}{h} = \frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h} = \frac{h(2x+h)}{h} = 2x + h$

2.3. Practice Problems.

- 1) Let $f(x) = x^2$ and $g(x) = \frac{1}{x}$ find $f(g(x))$
- 2) Let $f(x) = \frac{1}{x}$ and $g(x) = x^2$ find $f(g(x))$
- 3) Let $f(x) = \sqrt{3}x + 1$ and $g(x) = 3 - x$ find $f(g(x))$
- 4) Let $h(x) = 3 - x$ and $g(x) = \sqrt{3}x + 1$ find $h(g(x))$
- 5) Let $f(x) = \frac{1}{2x+1}$ and $g(x) = \log x$ find $f(g(x))$
- 6) Let $f(x) = \log x$ and $g(x) = \frac{1}{2x+1}$ find $f(g(x))$
- 7) Let $f(x) = (\frac{1}{4})^x$ and $g(x) = e^x$ find $f(g(x))$
- 8) Let $f(x) = e^x$ and $g(x) = (\frac{1}{4})^x$ find $f(g(x))$
- 9) Let $f(x) = x^2$ find $f(e^x)$
- 10) Let $g(x) = e^x$ and $h(x) = x^2$ find $g(h(x))$
- 11) Let $f(x) = x^2 + 2x + 1$ and $g(x) = x + 1$ find $f(g(x))$
- 12) Let $f(x) = x + 1$ find $f(x^2 + 2x + 1)$
- 13) Let $f(x) = x^2 + 1$ and $g(x) = 2$ find $f(g(x))$
- 14) Let $f(x) = 2$ and $g(x) = x^2 + 1$ find $f(g(x))$
- 15) Let $f(x) = 2^x$ and $g(x) = 2x$ find $f(g(x))$
- 16) Let $f(x) = 2x$ and $g(x) = 2^x$ find $f(g(x))$
- 17) Let $h(x) = \sin x$ and $g(x) = x^2$ find $h(g(x))$
- 18) Let $f(x) = x^2$ and $g(x) = \sin x$ find $f(g(x))$
- 19) Let $f(x) = 3x^2$ find $f(2x + 1)$
- 20) Let $f(x) = 2x + 1$ and $g(x) = 3x^2$ find $f(g(x))$

- 21) Let $f(x) = \sqrt{x^2 + 2}$ and $g(x) = \frac{1}{x}$ find $f(g(x))$
- 22) Let $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x^2 + 2}$ find $f(g(x))$
- 23) Let $f(x) = x^{\frac{2}{3}}$ and $g(x) = x^{\frac{3}{2}}$ find $f(g(x))$
- 24) Let $g(x) = x^{\frac{3}{2}}$ and $h(x) = x^{\frac{2}{3}}$ find $g(h(x))$
- 25) Let $f(x) = 2^{\frac{1}{x}}$ and $g(x) = \frac{1}{x}$ find $f(g(x))$
- 26) Let $f(x) = \frac{1}{x}$ and $g(x) = 2^{\frac{1}{x}}$ find $f(g(x))$
- 27) Let $f(x) = |x|$ and $g(x) = -x$ find $f(g(x))$
- 28) Let $f(x) = -x$ and $g(x) = |x|$ find $f(g(x))$
- 29) Let $f(x) = x^2$ and $g(x) = \frac{1}{x+1}$ find $f(g(x))$
- 30) Let $f(x) = \frac{1}{x+1}$ and $g(x) = x^2$ find $f(g(x))$
- 31) Let $f(x) = e^x$ and $g(x) = x + 1$ find $f(g(x))$
- 32) Let $f(x) = x + 1$ and $g(x) = e^x$ find $f(g(x))$
- 33) Let $f(x) = \sqrt{3}x$ and $h(x) = e^x$ find $f(h(x))$
- 34) Let $f(x) = e^x$ and $g(x) = \sqrt{3}x$ find $f(g(x))$
- 35) Let $f(x) = x^{\frac{3}{4}}$ and $g(x) = \frac{3}{4}x$ find $f(g(x))$
- 36) Let $f(x) = \frac{3}{4}x$ and $g(x) = x^{\frac{3}{4}}$ find $f(g(x))$
- 37) Let $h(x) = \ln x$ and $g(x) = e^{2x}$ find $h(g(x))$
- 38) Let $f(x) = e^{2x}$ and $g(x) = \ln x$ find $f(g(x))$
- 39) Let $f(x) = \log x$ and $g(x) = 10^{2x}$ find $f(g(x))$
- 40) Let $f(x) = 10^{2x}$ and $g(x) = \log x$ find $f(g(x))$
- 41) Let $f(x) = \log x$ find $f(e^x)$
- 42) Let $f(x) = e^x$ and $g(x) = \log x$ find $f(g(x))$
- 43) Let $f(x) = \log x$ and $g(x) = x^2$ find $f(g(x))$
- 44) Let $f(x) = x^2$ and $g(x) = \log x$ find $f(g(x))$
- 45) Let $f(x) = \log x$ and $g(x) = \sqrt{3}x$ find $f(g(x))$
- 46) Let $h(x) = \sqrt{3}x$ and $g(x) = \log x$ find $h(g(x))$
- 47) Let $f(x) = \frac{1}{2x+1}$ Find $f(x+1)$
- 48) Let $f(x) = \frac{1}{2x+1}$ Find $f(x^2)$
- 49) Let $f(x) = \frac{1}{2x+1}$ Find $f(x) + f(x+1)$
- 50) Let $f(x) = \frac{1}{2x+1}$ Find $f(x) + 1$

3. INVERSE FUNCTIONS

Given a function f , the inverse function for f is the function f^{-1} such that $(f \circ f^{-1}) = (f^{-1} \circ f) = x$. We note that in order for a function to have an inverse, it must be a 1-1 function, i.e., it must have the property that if $f(x)=f(y)$ then $x=y$. Otherwise the inverse map won't be a function as $f^{-1}(f(x))$ could be either x or y . The procedure for finding inverse functions of a function, $f(x)$ is as follows.

- 1: Replace the $f(x)$ in the equation by y .
- 2: Replace x and y in the equations, i.e., everywhere you see an ' x ' write a ' y ' and everywhere you see a ' y ' write an ' x '
- 3: Solve the equation for x
- 4: Replace y in the new equation for $f^{-1}(x)$

3.1. Example 1. Problem: Find the inverse of $f(x) = \frac{2x+2}{3x+10}$

Solution:

- 1: $y = \frac{2x+2}{3x+10}$
- 2: $x = \frac{2y+2}{3y+10}$
- 3: $x(3y + 10) = (2y + 2) \Rightarrow 3xy + 10x = 2y + 2 \Rightarrow 3xy - 2y = 2 - 10x \Rightarrow y(3x - 2) = 2 - 10x \Rightarrow y = \frac{2 - 10x}{3x - 2}$
- 4: $f^{-1}(x) = \frac{2 - 10x}{3x - 2}$

3.2. Example 2. Problem: Find the inverse of $f(x) = 3 * 2^x$

Solution:

- 1: $y = 3 * 2^x$
- 2: $x = 3 * 2^y$
- 3: $\log x = \log 3 * 2^y \Rightarrow \log x = \log 3 + y \log 2 \Rightarrow y = \frac{\log \frac{x}{3}}{\log 2}$
- 4: $f^{-1}(x) = \frac{\log \frac{x}{3}}{\log 2}$

3.3. Practice Problems. Find the inverse of:

- 1) $x + 1$
- 2) $x^2 + 1$
- 3) $\ln 2x - 1$
- 4) $\sqrt[3]{x}$
- 5) $\sqrt[3]{x} - 2$
- 6) $(\sqrt[3]{x} - 2)^3$
- 7) $(\sqrt[3]{x} - 2)^3 + 1$
- 8) $2x$
- 9) $(2x)^3$
- 10) $(2x)^3 - 1$
- 11) $(2x - 1)^3$
- 12) $\frac{2x+3}{3x+5}$
- 13) $2x^3 - 1$
- 14) $\frac{x^2+1}{4-x^2}$
- 15) $\frac{-x}{1-x}$
- 16) $\log(x)$
- 17) $\log(3x)$
- 18) $3\log(x)$
- 19) $\log(3x - 1)$
- 20) $\log\left(\frac{x}{3}\right)$
- 21) 100^x
- 22) $\log(10 + 10^x)$
- 23) $\frac{2x+1}{3x-1}$
- 24) $\log(2x - 4)$
- 25) $e^{(2x - 4)}$
- 26) e^{2^x}
- 27) 10^{2x+1}
- 28) $\frac{x+1}{x-1}$
- 29) $\frac{x-1}{x+1}$
- 30) $3x^5 - 4$
- 31) $(5 - \sqrt[3]{x})^3$
- 32) $4 + \log(1 + 3x^2)$
- 33) $10 * 10^x$

- 34) $\sqrt[3]{x^3 - 8}$
 35) $x^2 + 1$
 36) $x^3 - 1$
 37) $\frac{1}{x+1}$
 38) $(x + 1)^2$
 39) $\frac{1}{(x+1)^2}$
 40) e^{x+1}
 41) $\frac{e^{x+1}}{2}$
 42) $\frac{e^{x+1}}{2} + 1$
 43) $\left(\frac{e^{x+1}}{2}\right)^2$
 44) $\log(x + 1)$
 45) $\frac{\log(x+1)}{2}$
 46) $\left(\frac{\log(x+1)}{2}\right)^2$
 47) $\frac{\log(x+1)}{2} + 1$
 48) x
 49) $x^{2\pi}$
 50) $e^{2\pi x}$

4. ALGEBRA RULES

This section will review basic algebraic rules of real numbers,

- $a+(b+c)=(a+b)+c$ Associativity of addition
- $a+b = b+a$ Commutivity of addition
- $a(bc)=(ab)c$ Associativity of multiplication
- $ab = ba$ Commutivity of multiplication
- $a(b+c)=ab+ac$ Distributive property

These properties are used in to simply or expand expressions but more in general to manipulate equations. Some of the basic 'factoring' forms comes from these properties such as the multiplication of two binomial terms or the FOIL method.

$$(a+b)(c+d) = a(c+d) + b(c+d) = ac+ad + bc + bc.$$

By using this we can see that

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - ab - ba - b^2 = a^2 - 2ab - b^2$$

$$(a + b)(a - b) = a^2 + ba - ab - b^2 = a^2 - b^2$$

We note that $(a + b)^2 \neq a^2 + b^2$

When it comes to rational expressions, one must keep in mind of when one can 'cancel' or not. We have the following generic forms:

$$\frac{ab}{b} = a$$

where as

$$\frac{a+b}{b} \neq a$$

These problems are not asking for a 'solution' as in they don't want $x = \dots$ as the answer, for these problems, the answer is an equivalent algebraic expression. They may ask for you to expand or simply an expression, or they may ask you to pick equivalent expressions out of a list.

4.1. Example 1. Problem: Expand $(x + 5)(x^2 + 2)^2$

$$\text{Solution: } (x + 5)(x^4 + 4x^2 + 4) = x^5 + 4x^3 + 4x + 5x^4 + 20x^2 + 20 = x^5 + 5x^4 + 4x^3 + 20x^2 + 20$$

4.2. Example 2. Problem: Which of the following are equivalent to: $\frac{x+3}{9-x^2}$

$$\text{a) } \frac{3}{9-x} \quad \text{b) } \frac{1}{3-x} \quad \text{c) } \frac{x+1}{3-x^2} \quad \text{d) } \frac{1}{3x}$$

Solution: Only (b) is equivalent. as $\frac{x+3}{9-x^2} = \frac{x+3}{(3+x)(3-x)} = \frac{1}{3-x}$ and none of the other expressions are equivalent to (b).

4.3. Practice Problems. Expand the following

$$1) (x - 2)^2$$

$$2) (x - 2)^2(x + 1)$$

$$3) (x + 1)^2$$

$$4) (x + 1)^2(x - 2)$$

$$5) (x + 1)(x + 2)(x + 3)$$

$$6) (x - 2)^3$$

$$7) 5(2x + 1)^2(x - 2)$$

$$8) 7(x + 1)(x - 2)^2$$

- 9) $3(x^2 - 5)^2$
 10) $4(2x - 5)^2$

Write the following as a single rational function

- 11) $\frac{1}{x+2} - \frac{1}{2x+4}$
 12) $\frac{1}{x} + \frac{2}{x^2}$
 13) $x + \frac{1}{x}$
 14) $2 + \frac{1}{x}$
 15) $\frac{x+1}{x-1} + \frac{x-1}{x+1}$

Determine which of the following is equivalent to $8(x^2 - x)^7(2x - 1)$

- 16) $(8x^2 - 8x)^7(2x - 1)$
 17) $(x^2 - x)^7(16x - 8)$
 18) $16(x^2 - x)^7 - 8$
 19) $16x(x^2 - x)^7 - 1$
 20) $8x(x - 1)^7(2x - 1)$
 21) $8(x^2 - x)^6(x^2 - x)(2x - 1)$
 22) $16x(x^2 - x)^7 - 8(x^2 - x)^7$
 23) $16(x^2 - x)^7(x - \frac{1}{2})$
 24) $\frac{1}{8} \frac{(x^2 - x)^{-7}}{2x - 1}$
 25) $8(x - 1)^7(2x - 1)$

Determine which of the following is equivalent to $\frac{2}{4^x}$

- 26) $\frac{1}{2^x}$
 27) $2 * 4^{-x}$
 28) 8^{-x}
 29) 2^{1-2x}
 30) $(\frac{1}{2})^x$
 31) $\frac{4}{8^x}$
 32) $\frac{4^{-x}}{2^{-1}}$
 33) $4^{\frac{1}{2}-x}$
 34) $\frac{2}{4^x}$
 35) 2^{-x}

Determine which of the following is equivalent to $\sqrt{x^2 + x^{-2}}$

36) $(x^2 + x^{-2})^{\frac{1}{2}}$

37) $x + \frac{1}{x}$

38) $\frac{x+1}{x}$

39) 1

40) 0

Determine which of the following is equivalent to $\frac{(x+4)-(x-2)}{(x+4)^2}$

41) $\frac{-(x-2)}{x+4}$

42) $\frac{2}{(x+4)^2}$

43) $\frac{6}{(x+4)^2}$

44) $\frac{1}{(x+4)} - \frac{x-2}{(x+4)^2}$

45) $\frac{\sqrt{6}}{(x+4)}$

Determine which of the following is equivalent to $\sqrt{16^x}$

46) 8^x

47) $16^{\frac{1}{2}x}$

48) 4^x

49) $16^{\frac{2}{x}}$

50) 2^{2x}

5. EXPONENTS

Exponents follow the following rules:

- $a^m a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $a^0 = 1$
- $\sqrt[n]{a} = a^{\frac{1}{n}}$

Therefore we can use these rules to simplify expressions involving exponents.

5.1. **Example 1.** Problem: Write as a constant times x to a power: $\frac{5x^2}{\sqrt{3x^{-2}}}$

$$\text{Solution: } \frac{5x^2}{\sqrt{3x^{-2}}} = \frac{5x^2}{(3x^{-2})^{\frac{1}{2}}} = \frac{5x^2}{\sqrt{3}x^{-1}} = \frac{5x^2}{\frac{\sqrt{3}x^{-1}}{x}} = \frac{5x^2}{1} * \frac{x}{\sqrt{3}} = \frac{5}{\sqrt{3}}x^3$$

5.2. Practice Problems. Write the following as a constant times x to a power

- 1) $3x * x^2$
- 2) $(3x^2)^{-1}$
- 3) $\frac{3x^2}{6x^3}$
- 4) $(\frac{3x^2}{6x^3})^{-1}$
- 5) $(\frac{3x^2}{6x^{\frac{3}{4}}})^{-1}$
- 6) $2x\sqrt{x}$
- 7) $(2x\sqrt{x})^{-2}$
- 8) $x(\frac{1}{x})x^2(\frac{1}{x^3})(2x^2)$
- 9) $\sqrt[3]{\sqrt{x}}$
- 10) $\sqrt{\sqrt[3]{x}}$
- 11) $\sqrt[3]{\sqrt{\sqrt[3]{x}}}$
- 12) $\sqrt[3]{\sqrt{x}} * x^2$
- 13) $\frac{\sqrt[3]{\sqrt{x}}}{x}$
- 14) $\sqrt{x^2} * x^\pi$
- 15) $\sqrt{\sqrt[4]{\sqrt[3]{x}}}$
- 16) $\frac{\sqrt[3]{x^5}}{\sqrt{x^3}}$
- 17) $\frac{15x^2+30x^2}{5x}$
- 18) $\frac{2x^8}{x}$
- 19) $\frac{1}{3x}$
- 20) $\frac{1}{3x^2}$
- 21) $4x\sqrt[3]{x}$
- 22) $\frac{-16x^{-4}}{24x^2}\sqrt{x}$
- 23) $((((x)^2)^2)^2)$
- 24) $\sqrt{\sqrt{\sqrt{x}}}$

25) $\frac{-4}{x^{-1}}$

26) $\frac{\sqrt[3]{x}\sqrt{x^2}}{\sqrt{\sqrt[3]{x}}}$

Simplify the following to a single power of 2:

27) $2 * 4$

28) $2 * 4 * 8$

29) $2\sqrt{2}$

30) $\frac{1}{2\sqrt{2}}$

31) $\frac{8}{2\sqrt{2}}$

32) $\frac{16}{4}$

33) $16 * 32 * 64 * 128$

34) $\frac{128}{16} * \frac{64}{32} * 2$

35) $128\sqrt{2}$

36) $\frac{256}{\sqrt{2}}$

37) $4\sqrt{256}$

38) $\frac{\sqrt{256}}{\sqrt{64}}$

39) $(\sqrt{2})^2 \frac{1}{\sqrt[3]{2}}$

40) $(\sqrt{2}^3)(\frac{1}{\sqrt[3]{2}})(2^{50})$

Simplify the following to a single power of 3:

41) $\frac{1}{3}$

42) $\frac{1}{\sqrt{3}}$

43) $\sqrt[3]{3^5}$

44) $\sqrt[5]{3^3}$

45) $\sqrt{3} * \sqrt[3]{3} * 3$

46) $\frac{1}{3} * \frac{3}{9} * \frac{9}{27} * 27$

47) $\frac{1}{3^{-1}}$

48) $\sqrt{\frac{1}{3}}$

49) $\frac{27}{3\sqrt{3}}$

50) $\frac{9}{81}$

6. SOLVING EQUATIONS IN ONE VARIABLE

The first and most important thing to note here is that if $f(x)$ is a polynomial (and on the gateway they always are) we must have the same number of solutions as the highest power of x in the equation. Most notably about these problems though are solving quadratic equations i.e., equations of the form $ax^2+bx+c=0$. In the case where $a=1$, then sometimes we can get lucky and find numbers n and m such that $n+m=b$ and $nm=c$. In this instance we have that $x^2+bx+c=0 \Rightarrow (x+n)(x+m)=0$. In order to find solutions here we note that if the product of two numbers is zero then one of the numbers must be zero, so we obtain that either $x+n=0 \Rightarrow x=-n$ or $x+m=0 \Rightarrow x=-m$. So our solutions are $x=-n$ or $x=-m$, which is good because the highest power of x in this equations is 2 so we have the correct number of solutions. We note that we could only use because the right hand side of the equation is zero, it is not the case that $nm=6$ means that either $n=6$ or $m=6$. However this is a lucky situation, sometimes there are no such numbers n and m , or sometimes $a \neq 1$. In these cases we must use the quadratic formula which states that our solutions are $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ or $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$. We must keep in mind that if $b^2-4ac < 0$ then we are attempting to take the square root of a negative number, which is prohibited, so we must rewrite this using the number $i = \sqrt{-1}$. i.e., write $i\sqrt{5}$ instead of $\sqrt{-5}$. The next natural question is what happens when $b^2-4ac=0$? In this case the root $\frac{-b}{2a}$ is called a double root, and in so counts as two solutions. Another important note here is that we sometimes have to use the algebraic or exponent rules to move our equations into a form which we can solve.

6.1. **Example 1.** Problem: Solve for x : $x^3 + 5x^2 + 6x = 0$

Solution: First we note that we must have 3 solutions, we can obtain one quickly by factoring out an x to see that $x * (x^2 + 5x + 6) = 0$ means that either $x = 0$ which is one of the three solutions or $x^2 + 5x + 6 = 0$ which must give the other two as the highest power there is 2. We can see quickly that $(x+2)(x+3) = x^2 + 5x + 6$ so that the equation can be rewritten as $(x+2)(x+3) = 0$ which is zero if x is either -2 or -3, so we have solutions $x = 0, -2, -3$

6.2. **Example 2.** Problem: Solve for x : $\frac{-3(x^2-1)}{x} = 2$

Solution: We first must work this equation into a form we know, by multiplying by x we obtain $-3x^2 - 3 = 2x \Rightarrow -3x^2 + 2x - 3 = 0$ Now we should use the quadratic formula as no factoring stands out, so plugging in $a=-3, b=2, c=-3$ we have that $x = \frac{-2 \pm \sqrt{2^2 - 4 * (-3) * (-3)}}{2 * (-3)} \Rightarrow x = \frac{-2 \pm \sqrt{-32}}{-6} \Rightarrow x = \frac{2 \pm 5i}{6}$

6.3. **Example 3.** Problem: Solve for x : $x^2 + 6x + 9 = 0$

Solution: We note that $x^2 + 6x + 9 = (x+3)^2$ so we can solve $(x+3)^2 = 0 \Rightarrow x+3 = 0 \Rightarrow x = -3$ however note that this is a double root, so we are done. This could have been found using the quadratic formula, via $\frac{-6 \pm \sqrt{36 - 4 * 9}}{2} = \frac{-6}{2} = -3$

6.4. **Practice Problems.**

1) $x^3 + 2x^2 + 10x = x$

2) $(2x)^2 = 3$

3) $x^2 + x = 2$

4) $x^2 = \frac{-2x^2 + 5x - 1}{2}$

5) $2x^2 - 1 = 10x$

6) $5x = -6 - x^2$

7) $x^4 = 0$

8) $x^2 = 2x + 1$

9) $x^3 - 3x = 2x^2$

- 10) $x^2 - 12x = 14$
- 11) $2x^2 + 5 = 0$
- 12) $x^{18} = 1$
- 13) $x^2 - 2x + 1 = 0$
- 14) $x^2 = 25$
- 15) $x^4 - 5x^2 + 6 = 0$
- 16) $\frac{5x+1}{x} = 2x$
- 17) $\frac{x+2}{3x} = 5$
- 18) $\frac{x+1}{2x} = 10$
- 19) $\frac{3x+6}{x} = 3x$
- 20) $\frac{7x+12}{x} = x$
- 21) $\frac{20x}{x+1} = x + 5$
- 22) $\frac{12x+3}{x-1} = x + 7$
- 23) $\frac{3x+1}{x} = 2$
- 24) $\frac{x^2-3x+1}{x} = x + 1$
- 25) $\frac{x+14}{2x} = -5$
- 26) $\frac{2x+1}{3} = -5$
- 27) $\frac{-10x+2}{x+1} = x + 3$
- 28) $x^2 = 2x - 4$
- 29) $\sqrt{x} = 3x$
- 30) $\sqrt{x+1} = x + 2$
- 31) $x + 2 = 3x^2$
- 32) $x + 20 = x^2$
- 33) $x^3 + 2x^2 = 3x$
- 34) $\frac{7x+12}{x} = 3$
- 35) $\frac{2x+1}{x-2} = x + 1$
- 36) $\frac{10}{x+3} = x - 2$
- 37) $\frac{2x-2}{5x+4} = 5x + 2$
- 38) $\frac{12x-1}{x+2} = 3x + 3$
- 39) $\sqrt{x+1} = x - 2$
- 40) $\sqrt{x-1} = x$
- 41) $\frac{10x+4}{2x} = x - 3$

- 42) $\frac{3x-1}{2x+1} = 2x$
 43) $5x^2 + 2x = 2$
 44) $\frac{2x+1}{3} = x + 1$
 45) $\frac{3x+2}{x-1} = x + 2$
 46) $\frac{20x+2}{4x} = x - 2$
 47) $x^2 - 7x = -5$
 48) $2x^2 - 4 = 3x$
 49) $2x + 3 = 5x^2 - 7$
 50) $x^2 = -1$

7. LOGARITHMS

Logarithms follow the following properties: Logs in base 10

- $\log(A) = x \Leftrightarrow 10^x = A$
- $\log(AB) = \log(A) + \log(B)$
- $\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$
- $\log(A^p) = p\log(A)$
- $10^{\log(x)} = x$
- $\log(10^x) = x$
- $\log(1) = 0$

Logs in base e i.e., \ln follows the same rules as above except one must replace 10 with the number e. Using these rules we can to solve equations involving variables in the exponents.

7.1. **Example 1.** Problem: Solve for t: $25^{2t} = 5$

$$\text{Solution: } \ln 25^{2t} = \ln 5 \Rightarrow 2t * \ln 25 = \ln 5 \Rightarrow t = \frac{\ln 5}{2 * \ln 25}$$

7.2. **Example 2.** Problem: Solve for t: $\log_5 t = 20$

$$\text{Solution: } 10^{\log_5 t} = 10^{20} \Rightarrow 5t = 10^{20} \Rightarrow t = \frac{10^{20}}{5}$$

7.3. **Practice Problems.** Solve for x:

- 1) $\log(2x) = 4$
- 2) $3 * 5^x = 4 * 3^x$

- 3) $10^{2x+1} = 5$
- 4) $\log(3x) = 5$
- 5) $2\ln(x) = 12$
- 6) $\log(3x) - \log(5x) = 2\log(2x)$
- 7) $15 * e^x = 2 * 20^x$
- 8) $2e^{3x+10} = 15$
- 9) $\ln(x^2) - \ln(x) = 2$
- 10) $\log\left(\frac{5}{x}\right) = 40$
- 11) $\log(5x^2) = \log(3x) + 1$
- 12) $\ln(2x) = 10$
- 13) $3 * 8^x = 15 * 3^x$
- 14) $4^{\sqrt{x}+1} = 10$
- 15) $\log(20x) = 2$
- 16) $7\ln(x) = 9$
- 17) $2^{-2x+3} = 5$
- 18) $\log(3x) - 5\log(2x) = 1$
- 19) $9^{3x+5} = 3$
- 20) $\log(4x + 2) = 30$
- 21) $\ln(3x) = 10$
- 22) $3e^{3x+1} = 20e^2$
- 23) $e^{2x-2} = 11$
- 24) $\log(3x + 7) = \pi$
- 25) $\ln(2x + 9) = 17$
- 26) $2 * 5^x = 7 * 3^x$
- 27) $5 * 4^x = 3 * (12)^x$
- 28) $3 * 2^{x+1} = 5 * 7^x$
- 29) $\log(3x + 1) = 5$
- 30) $3 * (15)^x = 2 * 7^x$
- 31) $\ln(3x + 1) = \ln(2x)$
- 32) $e^{5x+1} = 20$
- 33) $e^{(3x+1)} = 50$
- 34) $\ln(20x + 2) = 13$
- 35) $10^{x^2} = 4$
- 36) $e^{x^2+1} = 10$

- 37) $10^{\sqrt{x}} = 4$
- 38) $\ln(2x + 1) = 5$
- 39) $e^{3x^2+1} = 10$
- 40) $\log(30x + 8) = 40$
- 41) $6 * 5^x = 20 * 4^x$
- 42) $\ln(5x + 2) = 20$
- 43) $e^{(2x+10)} = 5$
- 44) $2 * 7^x = 3 * 4^x$
- 45) $\log(12x + 3) = 5$
- 46) $3 * 6^x = 2 * 5^x$
- 47) $e^{(12x+10)} = 15$
- 48) $\ln(x) = 1$
- 49) $4^{(2x+2)} = 15$
- 50) $5^{(2x+1)} = 10$

8. SOLUTIONS

1.1.

- 1) slope = $\frac{1}{2}$ intercept = $\frac{-7}{2}$
- 2) $y = \frac{1}{2}x + \frac{3}{2}$
- 3) $y = -2x + 4$
- 4) slope = $\frac{-1}{2}$ intercept = $\frac{-1}{2}$
- 5) $y = \frac{-1}{2}x + \frac{1}{2}$
- 6) $y = 2x + 8$
- 7) slope = $\frac{-2}{5}$ intercept = $\frac{1}{5}$
- 8) $y = \frac{-2}{5}x + 3$
- 9) $y = \frac{5}{2}x$
- 10) slope = $\frac{2}{5}$ intercept = 0
- 11) $y = \frac{2}{5} - 1$
- 12) $y = \frac{-5}{2}x + 6$
- 13) slope = $\frac{-1}{2}$ intercept = $\frac{-1}{2}$
- 14) $y = \frac{-1}{2}x - \frac{1}{2}$
- 15) $y = 2x + 8$
- 16) slope = $\frac{-3}{2}$ intercept = 8

- 17) $y = \frac{-3}{2}x + 4$
- 18) $y = \frac{2}{3}x - 1$
- 19) slope = $\frac{1}{6}$ intercept = -3
- 20) $y = \frac{1}{6}x + 2$
- 21) $y = -6x + 39$
- 22) slope = $\frac{2}{3}$ intercept = $\frac{7}{3}$
- 23) $y = \frac{2}{3}x$
- 24) $y = \frac{-3}{2}x + 6$
- 25) slope = 1 intercept = 3
- 26) $y = x + 1$
- 27) $y = -x + 3$
- 28) $y = \frac{1}{4}x + \frac{7}{4}$
- 29) $y = \frac{-3}{2}x + \frac{15}{2}$
- 30) $y = \frac{2}{3}x + 1$
- 31) $y = 3x - 1$
- 32) $y = 3x - 2$
- 33) $y = \frac{-1}{3}x + \frac{4}{3}$
- 34) $y = x + \frac{1}{6}$
- 35) $y = -x + 2$
- 36) $y = x$
- 37) $y = 8x - 7$
- 38) $y = \frac{1}{2}x + 3$
- 39) $y = -2x + 8$
- 40) $y = 2x + 4$
- 41) $y = 0$
- 42) $x = 0$
- 43) $y = 2x - 3$
- 44) $y = \frac{-2}{3}x$
- 45) $y = \frac{3}{2}x$
- 46) $y = -x + 8$
- 47) $y = \frac{-5}{3}x + \frac{8}{3}$
- 48) $y = \frac{3}{5}x + \frac{2}{5}$
- 49) $y = 3$

$$50) y = \frac{-1}{3}x + \frac{5}{3}$$

1.2.

- 1) $\frac{1}{x^2}$
- 2) $\frac{1}{x^2}$
- 3) $\sqrt[3]{4-x}$
- 4) $3 - \sqrt[3]{x+1}$
- 5) $\frac{1}{2\log x + 1}$
- 6) $\log_{2x+1} \frac{1}{2}$
- 7) $\left(\frac{1}{4}\right)^{e^x}$
- 8) $e^{\left(\frac{1}{4}\right)^x}$
- 9) e^{2x}
- 10) e^{x^2}
- 11) $x^2 + 4x + 4$
- 12) $x^2 + 2x + 2$
- 13) 5
- 14) 2
- 15) 2^{2x}
- 16) 2^{x+1}
- 17) $\sin(x^2)$
- 18) $\sin^2 x$
- 19) $12x^2 + 12x + 3$
- 20) $6x^2 + 1$
- 21) $\sqrt{\frac{1}{x^2} + 2}$
- 22) $\frac{1}{\sqrt{x^2+2}}$
- 23) x
- 24) x
- 25) 2^x
- 26) $\frac{1}{2^{\frac{1}{x}}}$
- 27) x
- 28) $-|x|$
- 29) $\frac{1}{(x+1)^2}$
- 30) $\frac{1}{x^2+1}$

- 31) e^{x+1}
- 32) $e^x + 1$
- 33) $\sqrt[3]{e^x}$
- 34) $e^{\sqrt[3]{x}}$
- 35) $\left(\frac{3}{4}x\right)^{\frac{3}{4}}$
- 36) $\frac{3}{4}x^{\frac{3}{4}}$
- 37) $2x$
- 38) x^2
- 39) $2x$
- 40) x^2
- 41) $\log(e^x)$
- 42) $e^{\log x}$
- 43) $\log(x^2)$
- 44) $(\log(x))^2$
- 45) $\log(\sqrt[3]{x})$
- 46) $\sqrt[3]{\log(x)}$
- 47) $\frac{1}{2x+3}$
- 48) $\frac{1}{2x^2+1}$
- 49) $\frac{4x+4}{4x^2+5x+3}$
- 50) $\frac{2x+2}{2x+1}$

1.3.

- 1) $x - 1$
- 2) $\sqrt{x - 1}$
- 3) $\frac{e^x+1}{2}$
- 4) x^3
- 5) $(x + 2)^3$
- 6) $(\sqrt[3]{x - 1} + 2)^2$
- 7) $(\sqrt[3]{x - 1})^3$
- 8) $\frac{x}{2}$
- 9) $\frac{\sqrt[3]{x}}{2}$
- 10) $\frac{\sqrt[3]{x+1}}{2}$
- 11) $\frac{\sqrt[3]{x+1}}{2}$

- 12) $\frac{3-5x}{2x-2}$
- 13) $\sqrt[3]{\frac{x+1}{2}}$
- 14) $\sqrt{\frac{4x-1}{1+x}}$
- 15) $\frac{x}{x-1}$
- 16) 10^x
- 17) $\frac{10^x}{3}$
- 18) $10^{\frac{x}{3}}$
- 19) $\frac{10^x+1}{3}$
- 20) $3 * 10^x$
- 21) $\frac{\log(x)}{2}$
- 22) $\log(10^x - 10)$
- 23) $\frac{-x-1}{2-3x}$
- 24) $\frac{10^x+4}{2}$
- 25) $\frac{\ln(x)+4}{2}$
- 26) $\frac{\ln(x)}{2}$
- 27) $\frac{\log(x)-1}{2}$
- 28) $\frac{x+1}{x-1}$
- 29) $\frac{x-1}{x+1}$
- 30) $\sqrt[5]{\frac{x+4}{3}}$
- 31) $(-\sqrt[3]{x} - 5)^3$
- 32) $\sqrt{\frac{10^{x-4}-1}{3}}$
- 33) $\log(x) - 1$
- 34) $\sqrt[3]{x^3 - 8}$
- 35) $\sqrt{x - 1}$
- 36) $\sqrt[3]{x - 1}$
- 37) $\frac{1-x}{x}$
- 38) $\sqrt{x} - 1$
- 39) $\sqrt{\frac{1}{x}} - 1$
- 40) $\ln(x) - 1$
- 41) $\ln(2x) - 1$

42) $\ln(2x - 2) - 1$

43) $\ln(2\sqrt{x}) - 1$

44) $10^x - 1$

45) $10^{2x} - 1$

46) $10^{2\sqrt{x}} - 1$

47) $10^{2x-2} - 1$

48) x

49) $x^{\frac{1}{2}\pi}$

50) $\frac{\ln(x)}{2\pi}$

1.4.

1) $x^2 - 4x + 4$

2) $x^3 - 3x^2 + 4$

3) $x^3 + 2x + 1$

4) $x^3 - x^2 - x - 2$

5) $x^3 + 6x^2 + 11x + 6$

6) $x^3 - 4x^2 + 6x - 4$

7) $10x^3 - 20x^2 - 15x + 10$

8) $7x^3 - 7x^2 + 14x + 28$

9) $3x^4 - 30x^2 + 75$

10) $16x^2 - 80x + 100$

11) $\frac{1}{x+2}$

12) $\frac{x+2}{x^2}$

13) $\frac{x^2+1}{x}$

14) $\frac{2x+1}{x}$

15) $\frac{2x^2+2}{x^2-1}$

16) *No*

17) *Yes*

18) *No*

19) *No*

20) *No*

21) *Yes*

22) *Yes*

23) *Yes*

- 24) *No*
- 25) *No*
- 26) *No*
- 27) *Yes*
- 28) *No*
- 29) *Yes*
- 30) *No*
- 31) *No*
- 32) *Yes*
- 33) *Yes*
- 34) *No*
- 35) *No*
- 36) *Yes*
- 37) *No*
- 38) *No*
- 39) *No*
- 40) *No*
- 41) *No*
- 42) *No*
- 43) *Yes*
- 44) *Yes*
- 45) *No*
- 46) *No*
- 47) *Yes*
- 48) *Yes*
- 49) *No*
- 50) *Yes*

1.5.

- 1) $3x^3$
- 2) $\frac{1}{3}x^{-2}$
- 3) $\frac{1}{2}x^{-1}$
- 4) $2x$
- 5) $2x^{\frac{-5}{4}}$

- 6) $2x^{\frac{3}{2}}$
- 7) $\frac{1}{4}x^{-3}$
- 8) $2x$
- 9) $x^{\frac{1}{6}}$
- 10) $x^{\frac{1}{6}}$
- 11) $x^{\frac{1}{18}}$
- 12) $x^{\frac{13}{6}}$
- 13) $x^{\frac{-5}{6}}$
- 14) $x^{\pi+1}$
- 15) $x^{\frac{1}{24}}$
- 16) $x^{\frac{1}{6}}$
- 17) $9x$
- 18) $2x^7$
- 19) $\frac{1}{3}x^{-1}$
- 20) $\frac{1}{3}x^{-2}$
- 21) $4x^{\frac{4}{3}}$
- 22) $\frac{-2}{3}x^{\frac{-11}{2}}$
- 23) x^8
- 24) $x^{\frac{1}{8}}$
- 25) $-4x$
- 26) $x^{\frac{7}{6}}$
- 27) 2^3
- 28) 2^6
- 29) $2^{\frac{3}{2}}$
- 30) $2^{\frac{-3}{2}}$
- 31) $2^{\frac{3}{2}}$
- 32) 2^2
- 33) 2^{22}
- 34) 2^5
- 35) $2^{\frac{15}{2}}$
- 36) $2^{\frac{15}{2}}$
- 37) 2^6
- 38) 2

- 39) $2^{\frac{5}{6}}$
- 40) $2^{\frac{125}{3}}$
- 41) 3^{-1}
- 42) $3^{\frac{-1}{2}}$
- 43) $3^{\frac{5}{3}}$
- 44) $3^{\frac{3}{5}}$
- 45) $3^{\frac{11}{6}}$
- 46) 3^0
- 47) 3^{-1}
- 48) $3^{\frac{-1}{2}}$
- 49) $3^{\frac{3}{2}}$
- 50) 3^{-2}

1.6.

- 1) $x = 0, -1 \pm 2i\sqrt{2}$
- 2) $x = \pm \frac{1}{2}\sqrt{3}$
- 3) $x = 1, -2$
- 4) $x = 1, \frac{1}{4}$
- 5) $x = \frac{5}{2} \pm \frac{3}{2}\sqrt{3}$
- 6) $x = -2, -3$
- 7) $x = 0$
- 8) $x = 1 \pm \sqrt{2}$
- 9) $x = 0, 3, -1$
- 10) $x = 6 \pm 5\sqrt{2}$
- 11) $x = \pm \frac{i}{2}\sqrt{10}$
- 12) $x = \pm 1$
- 13) $x = 1$
- 14) $x = -5, 5$
- 15) $x = \pm\sqrt{2}, \pm\sqrt{3}$
- 16) $x = \frac{5}{4} \pm \frac{1}{4}\sqrt{33}$
- 17) $x = \frac{1}{7}$
- 18) $x = \frac{1}{19}$
- 19) $x = -1, 2$

- 20) $x = \frac{7}{2} \pm \frac{1}{2}\sqrt{97}$
- 21) $x = 7 \pm 2\sqrt{11}$
- 22) $x = 3 \pm \sqrt{19}$
- 23) $x = -1$
- 24) $x = \frac{1}{4}$
- 25) $x = \frac{-14}{11}$
- 26) $x = -8$
- 27) $x = -7 \pm -4\sqrt{3}$
- 28) $x = 1 \pm i\sqrt{3}$
- 29) $x = 0, \frac{1}{9}$
- 30) $x = \frac{-3}{2} \pm \frac{i}{2}\sqrt{3}$
- 31) $x = \frac{-2}{3}, 1$
- 32) $x = -4, 5$
- 33) $x = 0, 1, -3$
- 34) $x = -3$
- 35) $x = \frac{3}{2} \pm \frac{1}{2}\sqrt{21}$
- 36) $x = \frac{-1}{2} \pm \frac{1}{2}\sqrt{65}$
- 37) $x = \frac{-14}{25} \pm \frac{3i}{25}\sqrt{6}$
- 38) $x = \frac{1}{2} \pm \frac{5i}{6}\sqrt{3}$
- 39) $x = \frac{5}{2} \pm \frac{1}{2}\sqrt{13}$
- 40) $x = \frac{1}{2} \pm \frac{i}{2}\sqrt{3}$
- 41) $x = 4 \pm 3\sqrt{2}$
- 42) $x = \frac{1}{8} \pm \frac{i}{8}\sqrt{15}$
- 43) $x = \frac{-1}{5} \pm \frac{1}{5}\sqrt{11}$
- 44) $x = -2$
- 45) $x = 1 \pm \sqrt{5}$
- 46) $x = \frac{7}{2} \pm \frac{1}{2}\sqrt{51}$
- 47) $x = \frac{7}{2} \pm \frac{1}{2}\sqrt{29}$
- 48) $x = \frac{3}{4} \pm \frac{1}{4}\sqrt{41}$
- 49) $x = \frac{1}{5} \pm \frac{1}{5}\sqrt{51}$
- 50) $x = \pm i$

1.7.

- 1) $x = \frac{1}{2}10^4$
- 2) $x = \frac{\log(\frac{3}{4})}{\log(\frac{3}{5})}$
- 3) $x = \frac{\log(5)-1}{2}$
- 4) $x = \frac{1}{3}10^5$
- 5) $x = e^6$
- 6) $x = \sqrt{\frac{3}{20}}$
- 7) $x = \frac{\log(\frac{15}{2})}{\log(\frac{20}{e})}$
- 8) $x = \frac{1}{3}(\frac{\ln(15)}{\ln(2)} - 10)$
- 9) $x = e^2$
- 10) $x = \frac{5}{10^{40}}$
- 11) $x = 6$
- 12) $x = \frac{1}{2}e^{10}$
- 13) $x = \frac{\log(\frac{3}{15})}{\log(\frac{3}{8})}$
- 14) $x = (\frac{\ln(10)}{\ln(4)} - 1)^2$
- 15) $x = \frac{1}{20}10^2$
- 16) $x = e^{\frac{7}{9}}$
- 17) $x = \frac{-1}{2}(\frac{\ln(5)}{\ln(2)} - 3)$
- 18) $x = \sqrt[4]{\frac{3}{2^5}}$
- 19) $x = \frac{1}{3}(\frac{\ln(3)}{\ln(9)} - 5)$
- 20) $x = \frac{1}{4}(10^{30} - 2)$
- 21) $x = \frac{1}{3}(e^{10})$
- 22) $x = \frac{1}{3}(\ln(\frac{20}{3}) + 1)$
- 23) $x = \frac{1}{2}(\ln(11) + 2)$
- 24) $x = \frac{1}{3}(10^\pi - 7)$
- 25) $x = \frac{1}{2}(e^{17} - 9)$
- 26) $x = \frac{\log(\frac{2}{7})}{\log(\frac{3}{5})}$
- 27) $x = \frac{\log(\frac{5}{3})}{\log(3)}$

- 28) $x = \frac{\log(\frac{3}{4})}{\log(\frac{3}{5})}$
29) $x = \frac{1}{3}(10^5 - 1)$
30) $x = \frac{\log(\frac{3}{2})}{\log(\frac{7}{15})}$
31) $x = -1$
32) $x = \frac{1}{5}(\ln(20) - 1)$
33) $x = \frac{1}{3}(\ln(50) - 1)$
34) $x = \frac{1}{20}(e^{13} - 2)$
35) $x = \sqrt{\frac{\ln(4)}{\ln(10)}}$
36) $x = \sqrt{\ln(10) - 1}$
37) $x = \left(\frac{\ln(4)}{\ln(10)}\right)^2$
38) $x = \frac{1}{2}(e^5 - 1)$
39) $x = \sqrt{\frac{1}{3}(\ln(10) - 1)}$
40) $x = \frac{1}{30}(10^{40} - 8)$
41) $x = \frac{\log(\frac{3}{10})}{\log(\frac{4}{5})}$
42) $x = \frac{1}{5}(e^{20} - 2)$
43) $x = \frac{1}{2}(\ln(5) - 10)$
44) $x = \frac{\log(\frac{2}{3})}{\log(\frac{4}{7})}$
45) $x = \frac{1}{12}(e^5 - 3)$
46) $x = \frac{\log(\frac{3}{2})}{\log(\frac{5}{6})}$
47) $x = \frac{1}{12}(\ln(15) - 10)$
48) $x = 0$
49) $x = \frac{1}{2}\left(\frac{\ln(15)}{\ln(4)} - 2\right)$
50) $x = \frac{1}{2}\left(\frac{\ln(10)}{\ln(5)} - 1\right)$

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