Assignment 2 (T)

1. (Problem 2.1 from the textbook)
   Let $\Omega$ be a square with side 1. Show that
   $$\int_{\Omega} v^2 \, dx \leq \int_{\Omega} |\nabla v|^2 \, dx, \quad \forall v \in H^1_0(\Omega).$$

2. (Problem 2.5 from the textbook)
   Give a variational formulation of the inhomogeneous Neumann problem
   $$-\Delta u + u = f \quad \text{in } \Omega$$
   $$\frac{\partial u}{\partial n} = g \quad \text{on } \partial \Omega$$
   and check if the conditions (i)-(iv) of Section 2.1 are satisfied. Give a physical interpretation of such boundary condition.

3. (Problem 2.6 from the textbook)
   Give a variational formulation of the problem
   $$-\Delta u + u = f \quad \text{in } \Omega$$
   $$\gamma u + \frac{\partial u}{\partial n} = g \quad \text{on } \partial \Omega,$$
   where $\gamma$ is a constant. When are the conditions (i)-(iv) of Section 2.1 satisfied? Give an interpretation of such boundary condition (sometimes called Robin boundary conditions).

4. (Problem 2.7 from the textbook)