Assignment 6

1. (40 points) Let

\[ T(h) = \frac{h}{2} f(a) + h \sum_{i=1}^{n-1} f(a + ih) + \frac{h}{2} f(b), \]

where

\[ x_i = a + ih, \quad i = 0, \ldots, n, \quad h = \frac{(b-a)}{n}. \]

(a) Write a program that approximates \( \int_a^b f(x)dx \) by the composite trapezoidal rule \( T(h) \). Starting with \( h = \frac{(b-a)}{n} \) compute \( \frac{|T(h) - T(h/2)|}{|T(h/2)|} \) and reduce \( h \) by a factor of 2 until

\[ \frac{|T(h) - T(h/2)|}{|T(h/2)|} < 10^{-6}. \]

Your program should return a table with \( h/2, T(h/2), \frac{|T(h) - T(h/2)|}{|T(h/2)|} \) and it should return the total number of function evaluations \( f(x) \). Your program should reuse computed function values as much as possible.

(b) Apply your programs in i. to approximate the five integrals

\[
\int_0^3 \frac{x}{1 + x^2} \, dx = \frac{1}{2} \log(10), \\
\int_0^{0.95} \frac{1}{1 - x} \, dx = \log(20), \\
\int_0^1 \text{humps}(x) \, dx \approx 29.858325395509230, \quad \text{type help humps for more info.}
\]

2. (10 points)

Describe a method for solving the so-called Volterra integral equation

\[ \phi(x) + \int_a^x k(x, y)\phi(y)dy = f(x) \quad \forall x \in [a, b]. \]