Assignment 5

1. (10 points)
   Use `fzero` to try to find a zero of each of the following functions in the given interval. Do you see any interesting or unusual behavior?
   
   (a) \( \text{atan}(x) - \pi/3 \) on \([0, 5]\)
   (b) \( 1/(x - \pi) \) on \([0, 5]\)
   (c) \( \text{sign}(x - 2)\sqrt{|x - 2|} \) on \([1, 4]\)

2. (30 points) Here is a cubic polynomial with three closely spaced real roots.
   \[ p(x) = 816x^3 - 3835x^2 + 6000x - 3125 \]
   
   (a) What are the exact roots of \( p \)?
   (b) Plot \( p(x) \) for \( 1.43 < x < 1.71 \). Show the location of the three roots.
   (c) Starting with \( x_0 = 1.5 \), what does Newton’s method do?
   (d) Starting with the interval \([1, 2]\), what does bisection do?
   (e) What is `fzero(p, [1, 2])`? Why?

3. (20 points) Utilities must avoid freezing water mains. If we assume uniform soil conditions, the temperature \( T(x, t) \) at a distance \( x \) below the surface and time \( t \) after the beginning of a cold snap is given approximately by
   \[ \frac{T(x, t) - T_s}{T_i - T_s} = \text{erf} \left( \frac{x}{2\sqrt{\alpha t}} \right) \]
   Here \( T_s \) is the constant surface temperature during the cold period, \( T_i \) is the initial soil temperature before the cold snap, and \( \alpha \) is the thermal conductivity of the soil. If \( x \) is measured in meters and \( t \) in seconds, then \( \alpha = 0.138 \times 10^{-6} \text{m}^2/\text{s} \). Let \( T_i = 20°C, T_s = -15°C \), and recall that water freezes at \( 0°C \). Use `fzero` to determine how deep a water main should be buried so that it will not freeze until at least 60 days exposure under these conditions.