Assignment 2

1. (10 points)

In exact arithmetic
\[ x + (y + z) = (x + y) + z. \]

Show that this may be no longer true if we use floating point arithmetic. Specifically, assume a floating point number system with \( \beta = 10 \) and \( m = 4 \).

Construct floating point numbers \( \overline{x}, \overline{y}, \overline{z} \) such that
\[ \text{fl}(\overline{x} + \text{fl}(\overline{y} + \overline{z})) \neq \text{fl}(\text{fl}(\overline{x} + \overline{y}) + \overline{z}). \]

2. (20 points)

Show how to rewrite the following expressions to avoid catastrophic cancelation for the indicated arguments.

(a) \( \sqrt{x + 1 - 1}, x \approx 0 \),
(b) \( (1 - \cos x)/\sin x, x \approx 0 \).

3. (30 points)

Assume the earth is a sphere of radius \( r = 6370 \) km. Then its surface area is given by \( A = 4\pi r^2 \).

(a) Using four-digit floating point arithmetic (\( \beta = 10 \) and \( m = 4 \)) compute the surface area of the earth.
(b) Using the same formula and precision, compute the difference in surface area if the value for the radius is increased by 1 km.
(c) Since
\[ \frac{d}{dr} A = 8\pi r, \]
the change in surface area is approximated by \( 8\pi r\delta \), where \( \delta \) is the change in radius. Use this approximation and the floating point arithmetic in (a) to compute the difference in surface area if the value for the radius is increased by \( \delta = 1 \) km.
(d) Determine which answer, (b) or (c), is more accurate.
(e) Explain the results you obtain in (a)(c).
(f) Redo the computations in (a)(c) on MATLAB. How small must the change \( \delta \) in radius be for the same phenomenon to occur?