1. Factor 13, 17, and 51 into primes in \( \mathbb{Z}[i] \).

2. Factor \( 2 + 9i \) into primes in \( \mathbb{Z}[i] \). (Hint: factor its norm in \( \mathbb{Z} \) first.)

3. In \( \mathbb{Z} \), the prime factorizations \( 6 = 2 \cdot 3 = (-2)(-3) \) are essentially the same since the only difference is a sign change. Keeping in mind that \( \{\pm 1, \pm i\} \) plays the role in \( \mathbb{Z}[i] \) of \( \{\pm 1\} \) in \( \mathbb{Z} \), show the prime factorizations

\[
5 = (1 + 2i)(1 - 2i) = (2 + i)(2 - i).
\]

in \( \mathbb{Z}[i] \) are essentially the same.

4. Do the factorizations \( 10 = 2 \cdot 5 = (1 + 3i)(1 - 3i) \) show \( \mathbb{Z}[i] \) does not have unique prime factorization?

5. Let \( p \) be a prime in \( \mathbb{Z} \) and suppose \( p | (a^2 + b^2) \) where \( a \) and \( b \) are integers without a common factor. Deduce that \( p \) is not prime in \( \mathbb{Z}[i] \) by the same kind of argument used to prove Fermat’s 2-square theorem from class.