1. Starting with the 2-square representation
   \[ 261 = \left( \frac{1725}{221} \right)^2 + \left( \frac{3126}{221} \right)^2, \]
carry out the geometric argument from class until you get a 2-square representation 261 = \( a^2 + b^2 \) with \( a, b \in \mathbb{Z} \).

2. Determine which of the following fractions are sums of two squares in \( \mathbb{Q} \): \( \frac{9}{10}, \frac{62}{77}, \frac{369}{40} \).

3. Starting with the 3-square representation
   \[ 13 = \left( \frac{18}{11} \right)^2 + \left( \frac{15}{11} \right)^2 + \left( \frac{32}{11} \right)^2, \]
carry out the geometric argument from class until you get a 3-square representation 13 = \( a^2 + b^2 + c^2 \) with \( a, b, c \in \mathbb{Z} \).

4. Determine which of the following integers are sums of 3 squares in \( \mathbb{Z} \), using Legendre’s theorem: 124, 983, 2005.

5. In \( \mathbb{Q}[\sqrt{2}] \), compute \( \frac{1-9\sqrt{2}}{5+3\sqrt{2}} \) in the form \( a+b\sqrt{2} \) with \( a, b \in \mathbb{Q} \).

6. Just using modular arithmetic, show for \( a, b, c \in \mathbb{Z} \) that
   \[ a^2 + b^2 + c^2 \equiv 0 \mod 8 \implies a, b, c \text{ are all even}. \]

Conclude that for \( n \in \mathbb{Z}^+ \), if we write \( n = 4^k n' \) with \( k \geq 0 \) and \( n' \) not divisible by 4 (i.e., extract the largest power of 4 from \( n \)), then \( n \) is a sum of 3 squares in \( \mathbb{Z} \) if and only if \( n' \) is a sum of 3 squares in \( \mathbb{Z} \). (Don’t appeal to results in class about sums of 3 squares in \( \mathbb{Q} \)!

7. Show that the following conditions on a positive integer \( n \) are equivalent:
   1) \( n = x^2 + y^2 + z^2 \) for some \( x, y, z \in \mathbb{Z} \),
   2) \( x^2 + y^2 + z^2 - nw^2 = 0 \) has a solution in \( \mathbb{Z} \) other than \((0,0,0,0)\),
   3) \( x^2 + y^2 + z^2 = 0 \) has a solution in \( \mathbb{Q}[\sqrt{-n}] \) other than \((0,0,0)\).

8. For the following values of \( n \), find a solution to \(-1 = x^2 + y^2 \) in \( \mathbb{Q}[\sqrt{-n}] \): 1, 3, 5, 6, 10, 14.

9. From each of the equations
   \[ 83 = 3^2 + 5^2 + 7^2, \quad 83 = \left( \frac{7}{3} \right)^2 + \left( \frac{53}{15} \right)^2 + \left( \frac{121}{15} \right)^2, \]
derive a solution to \(-1 = x^2 + y^2 \) in \( \mathbb{Q}[\sqrt{-83}] \).

10. Generalize the formula for primitive Pythagorean triples in \( \mathbb{Z} \) to a formula for all “primitive” solutions to \( f(t)^2 + g(t)^2 = h(t)^2 \) in polynomials \( f(t), g(t), h(t) \).