ADVICE ON MATHEMATICAL WRITING

This handout lists some writing tips when you are dealing with mathematical text.

1. Notation

- Do not begin sentences with a symbol.
  Bad: x is positive, so it has a square root.
  Good: Since x is positive, it has a square root.

  Bad: Let n be an even number. $n = 2m$ for some $m \in \mathbf{Z}$.
  Good: Let n be an even number. Thus $n = 2m$ for some $m \in \mathbf{Z}$.
  Good: Let n be an even number, so $n = 2m$ for some $m \in \mathbf{Z}$.

  Bad: One solution is $f(x) = \sin x$. $f(x)$ is periodic.
  Good: One solution is $f(x) = \sin x$. In this case, $f(x)$ is periodic.

- Do not use unnecessary notations when writing the statement of a theorem (or some claim in the middle of a proof).
  Bad: Every differentiable function f is continuous.
  Good: Every differentiable function is continuous.
  Good: All differentiable functions are continuous.

- When introducing notation, make it fit the context. A lot of the time a choice of notation is just common sense.
  Bad: Let m be a prime.
  Good: Let p be a prime.

  Bad: Let X be a set, and pick an element of X, say t.
  Good: Let X be a set, and pick an element of X, say x.

  Bad: Pick two elements of the set X, say x and u.
  Good: Pick two elements of the set X, say x and y.
  Good: Pick two elements of the set X, say x_1 and x_2.
  Good: Pick two elements of the set X, say x and x'.

- Always define new notation (is it a number? a function? of what type?) and be clear about its logical standing.
  Very bad: Since n is composite, $n = ab$.
  Bad: Since n is composite, $n = ab$ for some integers a and b.
  Good: Since n is composite, $n = ab$ for some integers a and b greater than 1. [*Every* integer is a product, since $n = n \cdot 1$, so writing $n = ab$ alone introduces no constraint whatsoever.]

  Bad: If a polynomial $f(x)$ satisfies $f(n) \in \mathbf{Z}$, does $f(x)$ have integer coefficients?
  Good: If a polynomial $f(x)$ satisfies $f(n) \in \mathbf{Z}$ for every $n \in \mathbf{Z}$, does $f(x)$ have integer coefficients?

- Do not duplicate the meaning of a variable within the same proof.
  Bad: To show the sum of two even numbers is even, suppose a and b are even. Then $a = 2m$ and $b = 2m$, for some integer m. We have $a + b = 4m = 2(2m)$, which is even. [Notice this "proof" showed the sum of two even numbers is always a multiple of 4, which is nonsense.]
  Good: To show the sum of two even numbers is even, suppose a and b are even. Then $a = 2m$ and $b = 2n$, for some integers m and n. We have $a + b = 2m + 2n = 2(m + n)$, which is even.

- Avoid overloading meaning into notation.
  Bad: Let $x > 0 \in \mathbf{Z}$.
  Good: Let x be an integer, with $x > 0$.
  Good: Let x be a positive integer.

- **NEVER** use the logical symbols $\forall$, $\exists$, $\wedge$, $\vee$ when writing, *except* in a paper on logic. Write out what you mean in ordinary language.
  Bad: The conditions imply $a = 0 \wedge b = 1$.
  Good: The conditions imply $a = 0$ and $b = 1$.

  Bad: If $\exists$ a root of the polynomial then there is a linear factor.
  Good: If there is a root of the polynomial then there is a linear factor.

  Bad: If the functions agree at three points, they agree $\forall$ points.
  Good: If the functions agree at three points, they agree at all points.

- Avoid silly abbreviations, or the misuse of standard notations.
  Bad: When n is $\int$, $2n$ is an even number.
  Good: When n is integral, $2n$ is an even number.
  Good: When n is an integer, $2n$ is an even number.

  Bad: Let z be a $\mathbf{C}$.
  Good: Let z be a complex number.
  Good: Choose $z \in \mathbf{C}$.

2. Equations and expressions

- If an equation or expression is important (either for its own sake or because you will refer back to it later), display the equation on its own line. If you need to refer to it later, label it (as (1), (2), and so on) on the side. Of course, if you only need to make a reference to a displayed equation or expression immediately before or after it appears, you could avoid a label and say "by the above equation," *etc.*
  Bad: As a special case of the binomial theorem,
  $$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4.$$
  [suppose several lines of text are here]
  By the equation 8 lines up, we see...
  Good: As a special case of the binomial theorem,
  $$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4. \tag{2.1}$$
  [suppose several lines of text are here]
  By equation (2.1), we see...

- If a single computation involves several steps, especially more than two, present the steps in stacked form.

  Bad:

  $(x+1)^3 = (x+1)^2(x+1) = (x^2+2x+1)(x+1) = x^3+3x^2+3x+1.$

  Bad:

$$\begin{aligned}(x+1)^3 &= (x+1)^2(x+1)\\ (x+1)^3 &= (x^2+2x+1)(x+1)\\ (x+1)^3 &= x^3+3x^2+3x+1.\end{aligned}$$

  Good:

$$\begin{aligned}(x+1)^3 &= (x+1)^2(x+1)\\ &= (x^2+2x+1)(x+1)\\ &= x^3+3x^2+3x+1.\end{aligned}$$

- Equations do not stand by themselves. They appear as part of a sentence and should be punctuated accordingly! If an equation ends a sentence, place a period at the end of the line. If an equation appears in the middle of a sentence, use a comma after the equation if one would naturally pause there. Sometimes no punctuation is needed after the equation. The following three examples illustrate each possibility.

  Good: We call x_0 a critical point of f when f is differentiable and

$$f'(x_0) = 0.$$

  Good: When f is differentiable, and x_0 satisfies

$$f'(x_0) = 0,$$

  we call x_0 a critical point.

  Good: When f is differentiable, any x_0 where

$$f'(x_0) = 0$$

  is called a critical point.

  (That the equation was displayed separately in each case simply serves to highlight its importance to the reader. It could have been included within the main text, and punctuation rules of course apply in the same way.)

3. Parentheses and Commas

- Avoid pointless parentheses.

  Bad: $(x+y)(x-y) = (x^2-y^2)$.
  Good: $(x+y)(x-y) = x^2-y^2$.
  Bad: If 7 is a factor of the product $(a_1a_2\cdots a_n)$, then ...
  Good: If 7 is a factor of the product $a_1a_2\cdots a_n$, then ...
  Bad: The length is a factor of $(p-1)$.
  Good: The length is a factor of $p-1$.
  Good: $(a+b)^2-(a+c)^2 = b^2-c^2+2ab-2ac$.
  Good: $(a+b)^2-(a+c)^2 = (b^2-c^2)+2ab-2ac$. [This example is good *only* if the writer wants the reader to view b^2-c^2 as a single term of the right side.]

- Use parentheses to avoid confusing the meaning between a subtraction sign and a negative sign.

  Very bad: $(a+b)-c=-ac-bc$. [If you look at the right side, you can see the writer meant for the left side to be the product of $a+b$ and $-c$, but the left side instead looks like "a plus b minus c."

  Bad: $(a+b)\cdot -c=-ac-bc$.

  Good: $(a+b)(-c)=-ac-bc$.

- Commas are natural places to pause briefly, but not as fully as a period. If you read something in your head, you should be able to notice badly placed commas, either because no pause should occur or because a period should be there instead of a comma.

  Bad: The condition we want is, $a=2b$.

  Good: The condition we want is $a=2b$.

  Bad: The set is infinite, we pick a large finite subset of it.

  Good: The set is infinite. We pick a large finite subset of it.

- While "If ..., then..." is a common phrase, it is bad English to write "Let..., then...."

  Very Bad: Let n be an even number, then $n=2m$ for some $m \in \mathbf{Z}$.

  Good: Let n be an even number. Then $n=2m$ for some $m \in \mathbf{Z}$.

  Good: Let n be an even number, so $n=2m$ for some $m \in \mathbf{Z}$.

4. Use helpful words

- Tell the reader where you are going.

  Good: We will prove this by induction on n.

  Good: We will prove this by induction on the dimension.

  Good: We argue by contradiction.

  Good: Now we consider the converse direction.

  Good: But $f(x)$ is actually continuous. To see why, consider...

  Good: The inequality is strict. Indeed, if there was equality then...

- Use key words to show the reader how you are reasoning (since, because, on the other hand, observe, note), but vary your choice of words to avoid monotonous writing. This may require you to completely rewrite a paragraph.

  Bad: We proved, for any a, that if a^2 is even, then a is even. Now suppose a^8 is even. Then a^4 is even. Then a^2 is even. Then a is even.

  Good: We proved, for any a, that if a^2 is even, then a is even. Now suppose a^8 is even. Then, by successively applying the result we proved to a^4, a^2, and a, we see that a is even.

- Watch your spelling! If you aren't sure of the difference between "necessary" and "neccessary" or "discriminate" and "discriminant," look it up. (Canadian students may use their own flavour of spelling.)

  Use "it's" only to mean "it is". The word "its" is on the same footing as "his," to denote possession.

  Bad: It's clear that $f(x)$ has a real root since it's degree is odd.

  Bad: Its clear that $f(x)$ has a real root since its degree is odd.

  Good: It's clear that $f(x)$ has a real root since its degree is odd.

Good: Since $f(x)$ has odd degree, clearly it has a real root. [Write like this if you can't remember the difference between its and it's.]

5. Proofread

Before submitting your work, read it over again. If you don't think something is well written, rewrite it.

6. Exercises

Rewrite all of the following sentences to correct the errors. (These are all based on actual student writing.)

So we have that, $x = y$.
Let $a \in A$ then for some $b \in B$ we have $a < b$.
Consider the matrix M, it can be rewritten in the following way.
This number is also prime so it is in the set.
We have $x \in A$ this implies it is positive.
From the hint we know that $ab = c$, multiplying by a^{-1} gives $b = a^{-1}c$.
When p is an prime number, it is 2 or it is odd.
Assume that p is not prime, let $p|xy$ with $x \leq y$.
We also now that a is even.
Let $x \in S$ then we have $x > 0$.
There is some x not in S, else we would have $x > 0$.
We can see that sine the number is positive its a square.
Now assume that $H \subset G$, then we have that every element of H is in G.
All off these functions are differentiable.
Let $x \in H$, we need to show $x > 0$.
Let $\{x_1, x_2, \dots\}$ be a totally ordered sequence, the following argument shows it has an upper bound.
We know $(A + B)$ is invertible.
It's not a trivial result since, 0 and 1, the two obvious choices can fail to have the desired property.
To show that $x = y$. First, suppose $x > y$.
We need only two show that $x \leq 0$.
Assume that $a = b$, we will get a contradiction.
Since p is prime, $(p - 1) > 0$.
Let $n = 0$ then we can suppose $m > n$.
For $i > 0$ $a_i = 0$, which is a contradiction.
Consider x_n we will show that it is positive.