

ERRORS IN MATHEMATICAL WRITING

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1. INTRODUCTION

The best way to learn how to write well is to read. Of course, students not planning to go to graduate school in mathematics or other technical subjects are not in the habit of reading technical books outside of class. (Anyone who does intend to go to graduate school should consider doing this, however.) This handout won't teach you how to write well, but will show you some common errors in mathematical writing. In your homework, aim to avoid these errors. Perhaps you may even learn them well enough that you instinctively avoid them on exams.

2. NOTATION

- Do not begin sentences with a symbol.
 - Bad: x is positive, so it has a square root.
 - Good: Since x is positive, it has a square root.

 - Bad: Let n be an even number. $n = 2m$ for some $m \in \mathbf{Z}$.
 - Good: Let n be an even number. Thus $n = 2m$ for some $m \in \mathbf{Z}$.
 - Good: Let n be an even number, so $n = 2m$ for some $m \in \mathbf{Z}$.

 - Bad: One solution is $f(x) = \sin x$. $f(x)$ is periodic.
 - Good: One solution is $f(x) = \sin x$. In this case, $f(x)$ is periodic.
- Do not use unnecessary notations when writing the statement of a theorem (or some claim in the middle of a proof).
 - Bad: Every differentiable function f is continuous.
 - Good: Every differentiable function is continuous.
 - Good: All differentiable functions are continuous.
- When introducing notation, make it fit the context. In some cases this is a matter of tradition within the subject. (For instance, in complex analysis a typical complex variable is z , but for historical reasons a complex variable in papers on number theory is usually written as s . In some branches of mathematics, the integers modulo m appear as \mathbf{Z}_m , while $\mathbf{Z}/m\mathbf{Z}$ is preferred in other areas.) However, a lot of the time a choice of notation is just common sense.
 - Bad: Let m be a prime.
 - Good: Let p be a prime.

 - Bad: Let X be a set, and pick an element of X , say t .

Bad: Let X be a set, and pick an element of X , say x_1 .

Good: Let X be a set, and pick an element of X , say x .

Bad: Pick two elements of the set X , say x and u .

Good: Pick two elements of the set X , say x and y .

Good: Pick two elements of the set X , say x_1 and x_2 .

Good: Pick two elements of the set X , say x and x' .

- Always define new notation (is it a number? a function? of what type?) and be clear about its logical standing.

Very bad: Since n is composite, $n = ab$.

Bad: Since n is composite, $n = ab$ for some integers a and b .

Good: Since n is composite, $n = ab$ for some integers a and b greater than 1. [*Every* integer is a product, since $n = n \cdot 1$, so writing $n = ab$ alone introduces no constraint whatsoever.]

Bad: If a polynomial $f(x)$ satisfies $f(n) \in \mathbf{Z}$, does $f(x)$ have integer coefficients?

Good: If a polynomial $f(x)$ satisfies $f(n) \in \mathbf{Z}$ for every $n \in \mathbf{Z}$, does $f(x)$ have integer coefficients?

- Do not duplicate the meaning of a variable within the same proof.

Bad: To show the sum of two even numbers is even, suppose a and b are even. Then $a = 2m$ and $b = 2m$, for some integer m . We have $a + b = 4m = 2(2m)$, which is even. [Notice this “proof” showed the sum of two even numbers is always a multiple of 4, which is of course nonsense.]

Good: To show the sum of two even numbers is even, suppose a and b are even. Then $a = 2m$ and $b = 2n$, for some integers m and n . We have $a + b = 2m + 2n = 2(m + n)$, which is even.

- Avoid overloading meaning into notation.

Bad: Let $x > 0 \in \mathbf{Z}$.

Good: Let x be an integer, with $x > 0$.

Good: Let x be a positive integer.

- Never use the logical symbols \forall , \exists , \wedge , \vee when writing, except in a course on logic.

Bad: The conditions imply $a = 0 \wedge b = 1$.

Good: The conditions imply $a = 0$ and $b = 1$.

Bad: If the functions agree at three points, they agree \forall points.

Good: If the functions agree at three points, they agree at all points.

- Avoid silly abbreviations, or the misuse of standard notations.

Bad: When n is f , $2n$ is an even number.

Good: When n is integral, $2n$ is an even number.

Bad: Let z be a \mathbf{C} .

Good: Let z be a complex number.

Good: Choose $z \in \mathbf{C}$.

3. EQUATIONS AND EXPRESSIONS

- If an equation or expression is important, either for later reference or for emphasis, display it on its own line. If you need to refer to it later, label it (as (1), (2), and so on) on the side. Of course, if you only need to make a reference to a displayed equation or expression immediately before or after it appears, you could avoid a label and say “by the above equation,” *etc.*

Bad: As a special case of the binomial theorem,

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4.$$

[suppose several lines of text are here]

By the equation 8 lines up, we see. . .

Good: As a special case of the binomial theorem,

$$(3.1) \quad (x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4.$$

[suppose several lines of text are here]

By equation (3.1), we see. . .

- If a single computation involves several steps, especially more than two, present the steps in stacked form.

Bad:

$$(x + 1)^3 = (x + 1)^2(x + 1) = (x^2 + 2x + 1)(x + 1) = x^3 + 3x^2 + 3x + 1.$$

Bad:

$$\begin{aligned} (x + 1)^3 &= (x + 1)^2(x + 1) \\ (x + 1)^3 &= (x^2 + 2x + 1)(x + 1) \\ (x + 1)^3 &= x^3 + 3x^2 + 3x + 1. \end{aligned}$$

Good:

$$\begin{aligned} (x + 1)^3 &= (x + 1)^2(x + 1) \\ &= (x^2 + 2x + 1)(x + 1) \\ &= x^3 + 3x^2 + 3x + 1. \end{aligned}$$

- Equations do not stand by themselves. They appear as part of a sentence and should be punctuated accordingly. If an equation ends a sentence, place a period at the end of the line. If an equation appears in the middle of a sentence, use a comma after the equation if one would naturally pause there. Sometimes no punctuation is needed after the equation. The following three examples illustrate each possibility.

Good: We call x_0 a critical point of f when f is differentiable and

$$f'(x_0) = 0.$$

Good: When f is differentiable, and x_0 satisfies

$$f'(x_0) = 0,$$

we call x_0 a critical point.

Good: When f is differentiable, any x_0 where

$$f'(x_0) = 0$$

is called a critical point.

(That the equation was displayed separately in each case simply serves to highlight its importance to the reader. It could have been included within the main text, and punctuation rules of course apply in the same way.)

4. PARENTHESES

- Avoid pointless parentheses.
 - Bad: $(x + y)(x - y) = (x^2 - y^2)$.
 - Good: $(x + y)(x - y) = x^2 - y^2$.
 - Good: $(a + b)^2 - (a + c)^2 = b^2 - c^2 + 2ab - 2ac$.
 - Good: $(a + b)^2 - (a + c)^2 = (b^2 - c^2) + 2ab - 2ac$. [This example is good only if the writer wants the reader to view $b^2 - c^2$ as a special part of the right side.]
- Use parentheses to avoid confusing the meaning between a subtraction sign and a negative sign.
 - Very bad: $(a + b) - c = -ac - bc$.
 - Bad: $(a + b) \cdot -c = -ac - bc$.
 - Good: $(a + b)(-c) = -ac - bc$.

5. USE HELPFUL WORDS

- Tell the reader where you are going.
 - Good: We will prove this by induction on n .
 - Good: We will prove this by induction on the dimension.
 - Good: We argue by contradiction.
 - Good: Now we consider the converse direction.
 - Good: But $f(x)$ is actually continuous. To see why, consider...
 - Good: The inequality is strict. Indeed, if there was equality then...
- Use key words to show the reader how you are reasoning (since, because, on the other hand, observe, note), but vary your choice of words to avoid monotonous writing. This may require you to completely rewrite a paragraph.
 - Bad: We proved that if a^2 is even, then a is even. Suppose a^8 is even. Then a^4 is even. Then a^2 is even. Then a is even.

Good: We proved that if a^2 is even, then a is even. Suppose a^8 is even. Then, by successively applying the result to a^4 , a^2 , and a , we see that a is even.

- Watch your spelling! If you aren't sure of the difference between "necessary" and "neccessary" or "discriminate" and "discriminant," look it up. (Canadian students may use their own flavour of spelling.)

Use "it's" only to mean "it is". The word "its" is on the same footing as "his," to denote possession.

Bad: It's clear that $f(x)$ has a root since it's degree is odd.

Bad: Its clear that $f(x)$ has a root since its degree is odd.

Good: It's clear that $f(x)$ has a root since its degree is odd.

Good: Since $f(x)$ has odd degree, clearly it has a root. [Write like this if you can't remember the difference between its and it's.]

6. PROOFREAD

Before submitting your work, read it over again. If something is not well written, rewrite it. You might even find a mistake in your proof while proofreading it. (Why do you think this activity is called proofreading?)