

Part One: *Systems, Systems, Systems...*

For each of $\mathbf{A} = \begin{pmatrix} -2 & 1 \\ -13 & 4 \end{pmatrix}$, $\mathbf{A} = \begin{pmatrix} -2 & 1 \\ -9 & 4 \end{pmatrix}$, and $\mathbf{A} = \begin{pmatrix} -2 & 1 \\ -8 & 4 \end{pmatrix}$,

Study the linear autonomous system $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$ by completing the following steps.

1. Find the eigenvalue(s) and associated eigenvector(s).
2. Classify the equilibrium point $(0, 0)$ and determine whether it is stable or unstable.
3. Find the general solution to the system.
4. Sketch the phase portrait.

Then discuss the bifurcation value(s) of the one parameter system $\frac{d\mathbf{Y}}{dt} = \mathbf{A}_\alpha \mathbf{Y}$ where

$$\mathbf{A}_\alpha = \begin{pmatrix} -2 & 1 \\ \alpha & 4 \end{pmatrix}.$$