Consider the autonomous system

$$\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= 2x^2 + 2y \end{aligned}$$

with initial condition $(x(1), y(1)) = (0, 2)$

Problem 1: *(4 points) Use Euler's Method with step size* $\Delta t = \frac{1}{2}$ *to approximate* $x(2)$ *and* $y(2)$*.*

Solution: Writing $m_i = \left.\frac{dx}{dt}\right|_{t=t_i}$ and $n_i = \left.\frac{dy}{dt}\right|_{t=t_i}$, we complete the table

t_i	x_i	y_i	m_i	n_i
1	0	2	2	4
$\frac{3}{2}$	1	4	4	10
2	3	9	–	–

having made use of the formulas $x(t_{i+1}) = x(t_i) + m_i \cdot \Delta t$ and $y(t_{i+1}) = y(t_i) + n_i \cdot \Delta t$, i.e., $x_{new} = x_{old} + x\text{-}slope \cdot time$ and $y_{new} = y_{old} + y\text{-}slope \cdot time$.

Consequently, we estimate $x(2) \approx 3$ and $y(2) \approx 9$.

Problem 2: *(4 points) Are you assured there is a solution for the given initial value problem? Are you assured a solution exists at time* $t = 2$*? Are you assured that a solution, if it exists, is unique?*

Solution: Since the functions $f(x, y) = y$ and $g(x, y) = 2x^2 + 2y$ are both *nice* for all values of x, y, and t (i.e., continuously differentiable), the Existence Theorem assures a solution to the initial value problem on an interval $1 - \varepsilon < t < 1 + \varepsilon$ for some $\varepsilon > 0$. As the Existence Theorem does not preclude ε from being small (here, less than 1), it is possible that a solution is not (cannot) be defined at time $t = 2$.

The Uniqueness Theorem assures that any solution to the system with $(x(1), y(1)) = (0, 2)$ is unique.

Problem 3: *(2 points) Write something about the* `ButterflyEffect` *application that clearly demonstrates your usage of it.*

Solution: It offers views of the xy-plane, xz-plane, and yz-plane (one at a time). It allows the value of the parameter r to be altered. It graphs (butterfly-shaped) solutions to slightly different initial value problems simultaneously using green and purple. For some amount of time the solutions remain almost identical (so you see only green) but eventually the solutions diverge (se you see both green and purple).