Problem 1: (10 points) Consider the predator-prey system modeled by the equations

$$\begin{aligned} \frac{dR}{dt} &= 4R\left(1-\frac{R}{4}\right)-2RF \\ \frac{dF}{dt} &= -2F+RF. \end{aligned}$$

Part (a): *Find all equilibrium solutions of the system.*

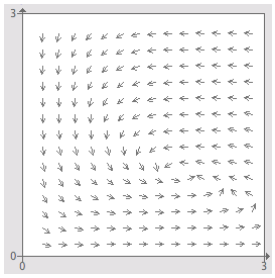
Solution: The equilibrium values occur when $\frac{dR}{dt}=0=\frac{dF}{dt}$, so we solve the equations $4R\left(1-\frac{R}{4}\right)-2RF=0$ and $-2F+RF=0$. Rewriting the first as $R(4-R-2F)=0$, we have either $R=0$ or $4-R-2F=0$.

If $R=0$, then $-2F+RF=0$ becomes $-2F=0$. Thus $F=0$, yielding the equilibrium $(0,0)$.

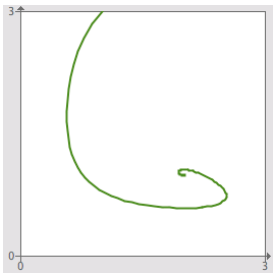
If $4-R-2F=0$, then $-2F+RF=0$ becomes $-2F+(4-2F)F=0$. Thus either $F=0$ or $F=1$. If $F=0$, then $4-R=0$ and so $R=4$, yielding the equilibrium $(4,0)$. If $F=1$, then $4-R-2=0$ and so $R=2$, yielding the equilibrium $(2,1)$.

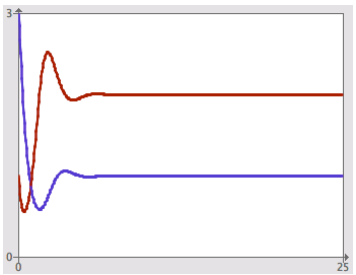
Part (b): *Draw part of the direction field for the system for $R\geq 0$ and $F\geq 0$. Note that a few (semi-random) accurately calculated and reasonably accurately drawn arrows are all that is requested.*

Solution: A computer generated direction field is pictured.



Part (c): *A solution curve for the system corresponding to the initial condition $(R(0),F(0))=(1,3)$ is pictured. Draw the $R(t)$ and $F(t)$ graphs for this solution curve.*





Solution: The $R(t)$ (red) and $F(t)$ (purple) graphs are pictured on the right.