MATH 2410 QUIZ 4 17 FEBRUARY 2009

Problem 1: (5 points) *Locate the bifurcation values for the one-parameter family of differential equations*

$$\frac{dy}{dt} = f_\alpha(y) = (y^2 - \alpha)(y^2 - 4)$$

and draw the bifurcation diagram.

Solution: The equilibrium values occur when $\frac{dy}{dt} = 0$. If $\alpha \geq 0$, these are at $y = \pm\sqrt{\alpha}$ and $y = \pm 2$; if $\alpha < 0$, these are at $y = \pm 2$. The bifurcation diagram (see figure) illustrates this.

There are thus two bifurcation values, namely $\alpha = 0$ and $\alpha = 4$. As α goes from less than zero to zero to greater than zero, the number of equilibria goes from two to three to four. As α goes from less than two to two to greater than two, the number of equilibria goes from four to two (where ± 2 and $\pm\sqrt{\alpha}$ overlap) to four.

Problem 2: (5 points) *Find the solution of the linear differential equation*

$$\frac{dy}{dt} = -y + 6e^{2t}$$

with $y(0) = 3$.

Solution: The associated homogeneous equation is $\frac{dy}{dt} = -y$. We recognize its general solution to be $y_h = ke^{-t}$.

In order to find a particular solution y_p to the nonhomogeneous equation, we rewrite it as $\frac{dy}{dt} + y = 6e^{2t}$ and guess $y_p = Ae^{2t}$ based on the form of the forcing function $6e^{2t}$. Then $\frac{dy_p}{dt} = 2Ae^{2t}$, and so

$$\frac{dy_p}{dt} + y_p = 2Ae^{2t} + Ae^{2t} = 3Ae^{2t} = 6e^{2t}$$

(if y_p is to be a solution to the nonhomogeneous equation). Thus $A = 2$, and so $y_p = 2e^{2t}$ is a particular solution.

The general solution to $\frac{dy}{dt} = -y + 6e^{2t}$ is thus $y = y_h + y_p = ke^{-t} + 2e^{2t}$. The initial condition $y(0) = 3$ implies $3 = k + 2$, and thus $k = 1$. The solution to the initial value problem is then

$$y = e^{-t} + 2e^{2t}.$$