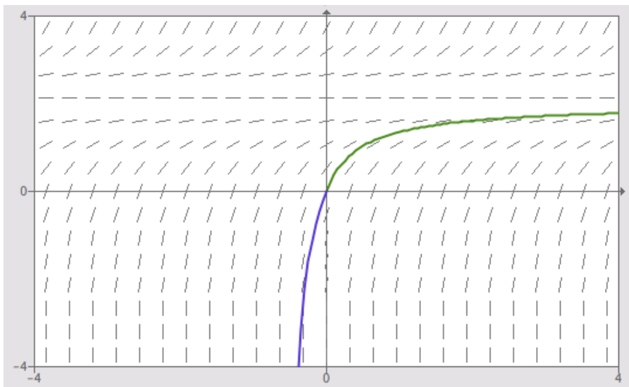
MATH 2410 QUIZ 2 3 FEBRUARY 2009

Problem 1: (5 points) *Consider the autonomous differential equation*

$$\frac{dS}{dt} = S^2 - 4S + 4.$$

Make a rough sketch of the slope field and the graph of the solution with the initial condition $S(0) = 0$.

Solution: HPGSolver was kind enough to produce the following sketch of the slope field and the solution with $S(0) = 0$.



Note that since the differential equation is autonomous (i.e., $\frac{dS}{dt} = f(S)$), the slope field is independent of t (i.e., looks the same shifted left and right).

Problem 2: (5 points) *Consider the initial value problem*

$$\frac{dS}{dt} = S^2 - 4S + 4; \qquad S(0) = 0.$$

Approximate $S(1)$ and $S(-1)$ using Euler's Method with a step size of $\Delta t = 1/2$.

Solution: Writing $f(S) = S^2 - 4S + 4$, we estimate values of S using

$$S_{new} = S_{old} + slope \cdot run.$$

For example, noting that $f(0) = 0^2 - 4(0) + 4 = 4$, we estimate

$$S(1/2) \approx S(0) + f(0) \cdot \Delta t = 0 + 4(1/2) = 2$$

and

$$S(-1/2) \approx S(0) + f(0) \cdot \Delta t = 0 + 4(-1/2) = -2.$$

Performing similar computations for $S(1)$ and $S(-1)$, we summarize the results in a table:

t	S	dS/dt
-1	-10	
-1/2	-2	16
0	0	4
1/2	2	0
1	2	

We therefore estimate $S(1) \approx 2$ and $S(-1) \approx -10$.