Problem 1: [BDH 1.5.12] (6 points) Verify (without finding the general solution) that

$$y_1(t) = \frac{1}{t-1} \qquad \text{and} \qquad y_2(t) = \frac{1}{t-2}$$

are solutions of the differential equation $\dfrac{dy}{dt} = -y^2$.

What can you say about solutions of $\dfrac{dy}{dt} = -y^2$ for which the initial condition satisfies the inequality $-1 < y(0) < -1/2$?

Address issues of both existence and uniqueness. Be as precise as possible and attempt to justify your answer completely.

Problem 2: [BDH 1.6.40] (4 points) Suppose you wish to model a population with an autonomous differential equation, i.e., an equation of the form $\frac{dP}{dt} = f(P)$, where $P(t)$ is the population at time t. Experiments have been performed on the population that give the following information:

- The population $P = 0$ remains constant.
- A population close to 0 will decrease.
- A population of $P = 20$ will increase.
- A population of $P > 100$ will decrease.

(a) Sketch the simplest possible phase line that agrees with the experimental information and classify the equilibrium points as sources, sinks, or nodes.

(b) Give a rough sketch of the function $f(P)$ for the phase line from Part (a).