Problem 1: *Consider a person presented with a list to be studied. Suppose that the rate at which they learn the list is proportional to the fraction of the list left to be learned. Further suppose, never having seen the list before, they start with zero knowledge.*

(a) (5 points) *Write down an initial value problem that models this scenario. Be sure to specify the meaning of any notation that you introduce.*

Solution: We let L denote the fraction of the list learned and let t denote the time elapsed since the person was presented with the list. Thus, for example, a value of $L = 1$ indicates the list has been entirely learned, while a value of $L = 0$ indicates none of the list has been learned. We note that $1 - L$ is the fraction of the list left to be learned and $\frac{dL}{dt}$ denotes the rate at which the list is being learned (or forgotten).

The second sentence of the problem statement implies the relationship

$$\frac{dL}{dt} = k(1 - L)$$

between these quantities, where k is the constant of proportionality. The third sentence of the problem statement implies $L(0) = 0$.

(b) (5 points) *If Asher's constant of proportionality is $\frac{1}{10}$ (he is a slow-learner...), solve for the fraction of the list learned as a function of time.*

Solution: We begin by finding the general solution to $\frac{dL}{dt} = \frac{1}{10}(1 - L)$ by separation of variables. Moving terms involving L to the left and terms involving t to the right, we have

$$\frac{1}{1 - L}\, dL = \frac{1}{10}\, dt.$$

Integrating both sides (the u-substitution $u = 1 - L$ is helpful on the left), we obtain

$$-\ln(1 - L) = \frac{1}{10}t + C,$$

noting that do not need absolute value signs around the quantity $1 - L$ since it will never be negative. Negating both sides, exponentiating both sides, replacing e^{-C} with a positive constant A, and solving for L, we find

$$L = L(t) = 1 - Ae^{-\frac{1}{10}t}.$$

Thus $0 = L(0) = 1 - Ae^{\frac{1}{10}(0)} = 1 - A$, and so $A = 1$. The solution to the initial value problem is thus

$$L = L(t) = 1 - e^{-\frac{1}{10}t}.$$