

PRACTICE FINAL EXAM

(1a) (7 pts) Solve the equation

$$dy/dt = y^2(t + 2)$$

with initial condition  $y(0) = 1$ .

(1b) (3 pts) For the above equation, sketch the slope field for representative  $(t, y)$  in the range  $-10 < t < 10$  and  $-10 < y < 10$ .

(2a) (10 pts) Consider the system of equations:

$$\begin{aligned}\frac{dx}{dt} &= y, \\ \frac{dy}{dt} &= -x^2 + t,\end{aligned}$$

with initial conditions  $x(0) = 1$  and  $y(0) = 2$ . Use Euler's method with  $\Delta t = 0.25$  to find the approximate solution for  $x$  and  $y$  at  $t = 0.5$ .

(2b) (5 pts) Consider  $u'' + u^2 = t$  with  $u(0) = 1$  and  $u'(0) = 2$ . Write it as a system of first order equation. Let energy  $E = (u')^2/2 + u^3/3$ . What is the approximate energy  $E$  at  $t = 0.5$  using the Euler's method and  $\Delta t = 0.25$ . (Hint: part (b) is related to part (a)).

(3) (12 pts) Consider the equation

$$\frac{dy}{dt} = y^2 - \mu y + 1.$$

At what values of the parameter  $\mu$  are there bifurcations? Draw phase lines for values of  $\mu$  just less than, just greater than, and at each of the bifurcations.

(4) (13 pts) Find the solution of  $dy/dt - 2y/t = t^2 e^t$  with initial condition  $y(1) = 0$ .

(5) (10 pts) Use the Laplace transform to solve:

$$\frac{dy}{dt} + y = 3e^{2t}$$

with initial condition  $y(0) = 1$ .

(6) (10 pts) Find the inverse Laplace transform of  $\frac{e^{-2s}}{(s+1)(2s+6)}$ .

(7) (10 pts) Find the general solution to

$$y'' + 2y' + y = 2 \sin 3t.$$

(8a) (6 pts) Given the matrix

$$B = \begin{pmatrix} 5 & -9 \\ 2 & -1 \end{pmatrix}$$

Find its (complex) eigenvalues and eigenvectors.

(8b) (6 pts) Given the matrix

$$D = \begin{pmatrix} 5 & 4 \\ 9 & 0 \end{pmatrix}$$

Find its (real) eigenvalues and eigenvectors.

(8c) (5 pts) Let  $A$  be a  $2 \times 2$  matrix with complex eigenvalues  $-2 + 3i$  and  $-2 - 3i$ . Assume the eigenvector corresponding to eigenvalue  $-2 + 3i$  is  $(i, 1)^T$ . Find the solution to the first order linear system  $d\mathbf{x}/dt = A\mathbf{x}$  with initial condition  $(1, 1)^T$  at  $t = 0$ .

(8d) (3 pts) Let  $E$  be a  $2 \times 2$  matrix with real eigenvalues 0 and  $-1$ . Assume the eigenvector corresponding to eigenvalue 0 is  $(1, 2)^T$ , and the eigenvector corresponding to eigenvalue  $-1$  is  $(1, 3)^T$ . Draw some representative solution trajectories in the phase plane for the equation  $d\mathbf{x}/dt = E\mathbf{x}$ .