

## PROBLEMS

**Miscellaneous Problems.** One of the difficulties in solving first order equations is that there are several methods of solution, each of which can be used on a certain type of equation. It may take some time to become proficient in matching solution methods with equations. The first 32 of the following problems are presented to give you some practice in identifying the method or methods applicable to a given equation. The remaining problems involve certain types of equations that can be solved by specialized methods.

In each of Problems 1 through 32 solve the given differential equation. If an initial condition is given, also find the solution that satisfies it.

1.  $\frac{dy}{dx} = \frac{x^3 - 2y}{x}$

2.  $(x + y) dx - (x - y) dy = 0$

3.  $\frac{dy}{dx} = \frac{2x + y}{3 + 3y^2 - x}, \quad y(0) = 0$

4.  $(x + e^y) dy - dx = 0$

5.  $\frac{dy}{dx} = -\frac{2xy + y^2 + 1}{x^2 + 2xy}$

6.  $x \frac{dy}{dx} + xy = 1 - y, \quad y(1) = 0$

7.  $\frac{dy}{dx} = \frac{x}{x^2y + y^3}$  Hint: Let  $u = x^2$ .

8.  $x \frac{dy}{dx} + 2y = \frac{\sin x}{x}, \quad y(2) = 1$

9.  $\frac{dy}{dx} = -\frac{2xy + 1}{x^2 + 2y}$

10.  $(3y^2 + 2xy) dx - (2xy + x^2) dy = 0$

11.  $(x^2 + y) dx + (x + e^y) dy = 0$

12.  $\frac{dy}{dx} + y = \frac{1}{1 + e^x}$

13.  $x dy - y dx = (xy)^{1/2} dx$

14.  $(x + y) dx + (x + 2y) dy = 0, \quad y(2) = 3$

15.  $(e^x + 1) \frac{dy}{dx} = y - ye^x$

16.  $\frac{dy}{dx} = \frac{x^2 + y^2}{x^2}$

17.  $\frac{dy}{dx} = e^{2x} + 3y$

18.  $(2y + 3x) dx = -x dy$

19.  $x dy - y dx = 2x^2y^2 dy, \quad y(1) = -2$

20.  $y' = e^{x+y}$

21.  $xy' = y + xe^{y/x}$

22.  $\frac{dy}{dx} = \frac{x^2 - 1}{y^2 + 1}, \quad y(-1) = 1$

23.  $xy' + y - y^2e^{2x} = 0$

24.  $2 \sin y \cos x dx + \cos y \sin x dy = 0$

25.  $\left(2\frac{x}{y} - \frac{y}{x^2 + y^2}\right) dx + \left(\frac{x}{x^2 + y^2} - \frac{x^2}{y^2}\right) dy = 0$

26.  $(2y + 1) dx + \left(\frac{x^2 - y}{x}\right) dy = 0$

27.  $(\cos 2y - \sin x) dx - 2 \tan x \sin 2y dy = 0$

28.  $\frac{dy}{dx} = \frac{3x^2 - 2y - y^3}{2x + 3xy^2}$

29.  $\frac{dy}{dx} = \frac{2y + \sqrt{x^2 - y^2}}{2x}$

30.  $\frac{dy}{dx} = \frac{y^3}{1 - 2xy^2}, \quad y(0) = 1$

31.  $(x^2y + xy - y) dx + (x^2y - 2x^2) dy = 0$

32.  $\frac{dy}{dx} = -\frac{3x^2y + y^2}{2x^3 + 3xy}, \quad y(1) = -2$