

1. Without using a calculator, find the numerical value of each of the following expressions: (a) $5!$ (b) ${}_{12}P_4$ (c) ${}_{12}C_4$

ANS: 120; 11,880; 495

2. In a survey of 100 coffee drinkers, it was found that 80 take sugar, 70 take cream, and 60 take both cream and sugar in their coffee.

(a) How many take sugar or cream?

(b) How many drink their coffee black, i.e., without sugar or cream?

ANS: 90; 10

3. It is estimated that 10% of Olympic athletes use steroids. The test currently being used to detect steroids is said to be 93% effective in correctly detecting steroids in users; it yields false positives in only 2% of tests. A country's best weightlifter tests positive. What is the probability that he actually takes steroids?

ANS: 93/111

4. A company car having a seating capacity of six is to be used by six employees who have formed a carpool. If only five of them can drive, how many possible seating arrangements are there for the group?

ANS: 600

5. A bin in a bargain outlet contains 100 blank cassette tapes, 12 of which are defective. A customer selects 5 of these blank cassette tapes at random.

(a) What is the probability that none of these five are defective?

(b) What is the probability that all of these five are defective?

(c) What is the probability that at least one of these five is defective?

(d) What is the probability that at least one of these five is not defective?

ANS: $\frac{{}_{88}C_5}{{}_{100}C_5}$; $\frac{{}_{12}C_5}{{}_{100}C_5}$; $1 - \frac{{}_{88}C_5}{{}_{100}C_5}$; $1 - \frac{{}_{12}C_5}{{}_{100}C_5}$

6. A bag contains 1 slip of paper with \$1000 written on it, 3 slips with \$100, 5 slips with \$20, 10 slips with \$5, and 2007 blank slips. For a \$1 fee, you may draw a random slip of paper and claim the amount on it as a prize. What is your expected value for the game?

ANS: $-\frac{576}{2026}$

7. Fraternal twinning is far more likely for women with a (dominant) gene on the X chromosome that causes hyper-ovulation. A man cannot affect whether his partner will hyper-ovulate, so males can only affect fraternal twinning by passing on this gene to their offspring.

(a) If Mrs. Delavan's mother's family has no known instances of fraternal twinning (infer Mrs. Delavan's mother does not have the dominant gene) but Mrs. Delavan's father's family has a high instance of fraternal twinning (infer Mrs. Delavan's father is a carrier of the dominant gene), what is the probability that Mrs. Delavan has the hyper-ovulation gene [given that she is female]?

(b) If in addition Mr. Delavan's mother's family has instances of twinning only on his maternal grandmother's side (infer Mr. Delavan's mother has both the dominant and recessive gene) but nothing is known about Mr. Delavan's father's family, what is the probability that a baby Delavan will have or be a carrier for the hyper-ovulation gene? What if the baby is known to be male? What if the baby is known to be female?

Fraternal twins occur in about 12 out of every 1000 births, while identical twins occur in about 3 out of every 1000 births. Fraternal twins can be of the same or different sex, while identical twins must be of the same sex.

(c) If Mr. and Mrs. Delavan know they are having twins but nothing else, what is the probability that they are having identical twins?

(d) If Mr. and Mrs. Delavan know they are having twins of the same sex but nothing else, what is the probability that they are having identical twins?

ANS: $1; \frac{5}{8}, \frac{1}{2}, \frac{3}{4}; \frac{1}{5}; \frac{1}{3}$