

Problem 1 (5 points): Find the number of ways to draw a one pair poker hand, i.e., a five card hand with two cards of the same rank and three other cards of three other different ranks.

Be sure to explain how you obtained your answer.

Solution: We observe that there are ${}_{13}C_1$ possible ways to choose the rank of the pair (the two cards of the same rank) and then ${}_{12}C_3$ possible ways to choose the ranks for the three other cards. Having chosen the ranks, there are ${}_4C_2$ possible ways to choose the suits of the pair, and ${}_4C_1$ possible ways to choose the suit of each of the three other cards.

There are thus

$$\begin{aligned} ({}_{13}C_1) ({}_{12}C_3) ({}_4C_2) ({}_4C_1) ({}_4C_1) ({}_4C_1) &= \left(\frac{13!}{1! 12!} \right) \left(\frac{12!}{3! 9!} \right) \left(\frac{4!}{2! 2!} \right) \left(\frac{4!}{1! 3!} \right)^3 \\ &= \left(\frac{13}{1} \right) \left(\frac{12 \times 11 \times 10}{3 \times 2 \times 1} \right) \left(\frac{4 \times 3}{2 \times 1} \right) \left(\frac{4}{1} \right)^3 \\ &= 1,098,240 \end{aligned}$$

ways to draw a one pair poker hand.

Problem 2 (5 points): [GH 4.6.28] A meat processor has packed 50 packages of meat, 5 of which contain contaminated meat. If an inspector checks only 4 of the 50 packages, what is the probability that she will not check any of the contaminated packages?

Solution: There are ${}_{50}C_4$ ways of choosing four packages to inspect. Of these, there are ${}_{45}C_4$ ways of choosing four packages from the forty-five uncontaminated packages. The probability of the inspector not checking any of the contaminated packages is thus

$$\begin{aligned} \text{Prob} &= \frac{{}_{45}C_4}{{}_{50}C_4} = \frac{\frac{45!}{4! 41!}}{\frac{50!}{4! 46!}} = \frac{45!}{4! 41!} \frac{4! 46!}{50!} \\ &= \frac{45!}{41!} \frac{46!}{50!} \\ &= \frac{45 \times 44 \times 43 \times 42}{1} \frac{1}{50 \times 49 \times 48 \times 47} \\ &= \frac{4257}{6580} \\ &\approx .6470. \end{aligned}$$