

The preference rankings of a class of 21 college students for their favorite board game are as follows:

	Number of Votes					
	1	4	6	2	5	3
Monopoly	1	1	2	3	2	3
Risk	2	3	1	1	3	2
Scrabble	3	2	3	2	1	1

**Part A (2 points):** Which game (if any) wins a plurality of the vote?

**Solution:** Five students ( $1 + 4$ ) ranked Monopoly as their top choice, eight students ( $6 + 2$ ) ranked Risk as their top choice, and eight students ( $5 + 3$ ) ranked Scrabble as their top choice, so there will be a tie between Risk and Scrabble if the winner is decided by the plurality method.

**Part B (2 points):** Which game (if any) wins a plurality election with a runoff between the top two finishers?

**Solution:** Risk and Scrabble are the top two finishers. During the runoff, Risk will receive nine votes ( $1 + 6 + 2$ ) whereas Scrabble will receive twelve votes ( $4 + 5 + 3$ ). Thus Scrabble will be the winner if it is decided by a plurality election with a runoff between the top two finishers.

**Part C (2 points):** Which game (if any) has the top Borda count?

**Solution:** The Borda count of Monopoly will be

$$1(3) + 4(3) + 6(2) + 2(1) + 5(2) + 3(1) = 42,$$

the Borda count of Risk will be

$$1(2) + 4(1) + 6(3) + 2(3) + 5(1) + 3(2) = 41,$$

and the Borda count of Scrabble will be

$$1(1) + 4(2) + 6(1) + 2(2) + 5(3) + 3(3) = 43$$

Thus Scrabble will be the winner if it is decided by the Borda method.

	Number of Votes					
	1	4	6	2	5	3
Monopoly	1	1	2	3	2	3
Risk	2	3	1	1	3	2
Scrabble	3	2	3	2	1	1

**Part D (2 points):** *The six voters who ranked Risk first, Monopoly second, and Scrabble third are unhappy with the Borda count. Could they obtain a favorable result in an election decided by the Borda method by voting strategically if the other members voted as shown in the table? If so, how? If not, why not?*

**Solution:** Yes, these voters could obtain a favorable result by placing Monopoly as their first choice, Risk as their second choice, and Scrabble as their third choice. In doing so, they create the following preference rankings:

	Number of Votes					
	1	4	6	2	5	3
Monopoly	1	1	1	3	2	3
Risk	2	3	2	1	3	2
Scrabble	3	2	3	2	1	1

With these new preference rankings, the Borda counts of Monopoly, Risk, and Scrabble will be

$$\text{Monopoly} : 1(3) + 4(3) + 6(3) + 2(1) + 5(2) + 3(1) = 48,$$

$$\text{Risk} : 1(2) + 4(1) + 6(2) + 2(3) + 5(1) + 3(2) = 35,$$

$$\text{Scrabble} : 1(1) + 4(2) + 6(1) + 2(2) + 5(3) + 3(3) = 43.$$

The game with the highest Borda count is then Monopoly, which is a more favorable result for these six students (with their true, original preferences).

**Question:** *Can these six students cause Risk to be the winner by voting strategically? Why or why not?*

**Part E (2 points):** *Eighteen (18) new students join the class. What percentage of these voters would Monopoly need to ensure a plurality? Note: Use the original rankings.*

**Solution:** Letting  $x$  represent the number of these voters that place Monopoly as their highest preference, there are  $18 - x$  voters that can place Risk or Scrabble as their highest preference. If Monopoly is to win the plurality election, it would need

$$5 + x > 8 + 18 - x.$$

Solving this for  $x$ , we find  $2x > 21$ , or  $x > 10.5$ . Thus Monopoly would need 11 of these 18 votes, which is roughly 61.11%. Note that it does not matter if the  $18 - x$  voters choose Risk or Scrabble (as long as they all choose the same) since both start with 8 votes.