

The Sixteenth International Workshop on Operator  
Theory and Applications  
University of Connecticut, July 24-27, 2005

Program and Abstracts

# PROGRAM SCHEDULE

## SATURDAY, JULY 23rd

3:00 - 8:00	Registration & Check-in;	Nathan Hale Inn Lobby
7:00 - 10:00	Welcome reception;	Rome Commons Ballroom

## SUNDAY, JULY 24th

	<b>Room 106</b> , School of Business bldg.	
8:30 - 8:40	opening remarks;	

Plenary talks,

Chair: William Helton

8:40 - 9:30	<b>Israel Gohberg</b> <i>Resultants for Matrix Polynomials and Applications</i>
9:30 - 10:20	<b>M.A.Kaashoek</b> <i>The continuous analogue of the resultant for entire matrix functions</i>
	coffee break

A Semiplenary talk,

Chair: M.A.Kaashoek

10:40 - 11:20	<b>Joseph Ball</b> <i>Multidimensional system theory, Lax-Phillips scattering and multi-variable operator theory: the polydisk setting</i>
---------------	---

11:25 – 11:55	<b>Leiba Rodman</b> <i>Convex Invertible Sets of Matrices</i>
11:55 – 12:25	<b>Eduard Tsekanovskii</b> <i>A general realization theorem for Herglotz-Nevanlinna matrix-valued functions</i>
	Lunch
1:45 – 2:15	<b>Miron Bekker</b> <i>Tangential Interpolation Problems for a Class of Automorphic Matrix-Functions</i>
2:15 – 2:45	<b>Lawrence Fialkow</b> <i>Solution of the extremal truncated moment problem</i>
2:45 – 3:15	<b>Sanne ter Horst</b> <i>All solutions to the relaxed commutant lifting problem</i>
3:15 – 3:45	<b>Michael Jury</b> <i>Matrix products and interpolation in reproducing kernel Hilbert spaces</i>
	coffee break

## Contributed talks

	<b>Room 106</b> Chair: Hugo Woerdeman	<b>Room 127</b> Chair: I.Spitkovsky
4:05 – 4:35	<b>Martin Klaus</b> <i>Multiple eigenvalues and spectral singularities in Zakharov-Shabat systems</i>	<b>Stephan Garcia</b> <i>Complex symmetric operators</i>
4:35 – 5:05	<b>Michael Lauzon</b> <i>Scalar and vector Muckenhoupt weights</i>	<b>Artem Pulemyotov</b> <i>Image of a Jacobi field</i>
5:05 – 5:35	<b>Saida Sultanic</b> <i>Commutant Lifting Theorem for the Bergman Space</i>	<b>Alexei Karlovich</b> <i>Higher order asymptotics of Toeplitz determinants with symbols in weighted Wiener algebras</i>

<b>6:00-8:00, BBQ party, Rome Commons Patio</b>
---

## Monday, JULY 25th

Plenary talks,

Chair: Joseph Ball

	<b>Room 106</b> , School of Business bldg.
8:45 – 9:35	<b>William Helton</b> <i>Convexity and Analysis of Noncommutative Polynomials</i>
9:35 – 10:25	<b>James Rovnyak</b> <i>Inverse problems for canonical differential equations with singularities</i>
	coffee break

A Semiplenary talk,

Chair: James Rovnyak

10:45 – 11:25	<b>L.A. Sakhnovich</b> <i>Integrable operators</i>
---------------	---

A Special Session on Interpolation,

Organizers: V.Bolotnikov, A.Kheifets, L.Rodman

11:25 – 11:55	<b>Aleksander Kheifets</b> <i>On Boundary Interpolation</i>
11:55 – 12:25	<b>Vladimir Bolotnikov</b> <i>Nevanlinna-Pick boundary interpolation for generalized Schur functions</i>
	Lunch / <b>Steering Committee meeting</b>
1:45 – 2:15	<b>M. A. Kaashoek</b> <i>The band method and the Nehari-Takagi problem</i>
2:15 – 2:45	<b>Greg Knese</b> <i>A Schwarz Lemma on the Polydisk</i>
2:45 – 3:15	<b>Michael Dritschel</b> <i>Interpolation on Semigroupoid Algebras</i>
3:15 – 3:45	<b>Mihaly Bakonyi</b> <i>Extensions of Positive Functions on Free Groups</i>
	coffee break

Contributed talks

	<b>Room 106</b> Chair: M.Dritschel	<b>Room 127</b> Chair: E.E.Tyrtysnikov
4:05 – 4:35	<b>Patrick Pan</b> <i>Separating Operators by Linear Functionals</i>	<b>Dana Clahane</b> <i>Norm equivalence of and composition operators between Lipschitz/Bloch spaces of the unit ball</i>
4:35 – 5:05	<b>David Opela</b> <i>Extension and Dilation Theorems for Operators Satisfying Relations</i>	<b>Michael Neumann</b> <i>Reduced Rank Nonnegative Matrix Factorization for Symmetric Nonnegative Matrices</i>
5:05 – 5:35	<b>Ilya Krishtal</b> <i>Linear Relations and Operator Semigroups</i>	<b>Tavan Trent</b> <i>A Vector-valued Corona Type Theorem on the Polydisk</i>

**Tuesday, JULY 26th**

Plenary talks,

Chair: Lothar Reichel

	<b>Room 106</b> , School of Busines bldg.
8:45 – 9:35	<b>Joachim Rosenthal</b> <i>Building Public Key Crypto-Systems from Operators over Semi-Rings</i>
9:35 – 10:25	<b>Paul Van Dooren</b> <i>On computing the complex passivity radius</i>
	coffee break

Semiplenary talks,

Chair: Paul Van Dooren

10:45 – 11:25	<b>Lothar Reichel</b> <i>Tikhonov Regularization of Large-Scale Problems</i>
11:25 – 12:05	<b>Eugene Tyrtysnikov</b> <i>Separability concepts and multilevel matrices</i>
	Lunch

A Special Session on Semiseparable matrices,  
Organizers: Yu.Eidelman and M. Van Barel

1:30 – 2:00	<b>Israel Koltracht</b> <i>Fast algorithms for semiseparable matrices and kernels</i>
2:00 – 2:30	<b>Yuli Eidelman</b> <i>The unitary completion and QR iterations for a class of structured matrices</i>
2:30 – 3:00	<b>Marc van Barel</b> <i>The Lanczos reduction to semiseparable matrices and applications in computing dominant subspaces</i>
3:00 – 3:30	<b>Raf Vandebril</b> <i>A Levinson-like solver for semiseparable plus band matrices</i>
	coffee break
3:50 – 4:20	<b>Tom Bella</b> <i>A Bjork-Pereyra-type algorithm for the Szego-Vandermonde matrices</i>
4:20 – 4:50	<b>Dario Fasino</b> <i>Perturbative analysis of reductions to diagonal-plus-semiseparable form</i>
4:50 – 5:20	<b>Vadim Olshevsky</b> <i>Eigenvalue problems and quasiseparable matrices</i>
5:20 – 5:50	<b>Ming Gu</b> <i>A Fast and Stable Polynomial Rootfinder</i>

**7:00-9:00, Banquet, Nathan Hale Inn**

**Wednesday, JULY 27th**

A Plenary talk,

Chair: Aad Dijksma

	<b>Room 106</b> , School of Business bldg.
8:30 – 9:20	<b>Hugo Woerdeman</b> <i>The Operator Valued Autoregressive Filter Problem and the Suboptimal Nehari Problem in Two Variables</i>

Semiplenary talks,

Chair: L.A. Sakhnovich

9:20 – 10:00	<b>Aad Dijksma</b> <i>n the generalized Schur transform of generalized Schur and Nevanlinna functions: operator realizations, interpolation, and factorization</i>
10:00 – 10:40	<b>Ilya Spitkovsky</b> <i>Factorization in Wiener algebras on ordered abelian groups</i>
	break

Contributed talks,

Chair: Israel Koltracht

11:00 – 11:30	<b>Yuri Lyubich</b> <i>The Sylvester-Ramanujan equations and related topics</i>
11:30 – 12:00	<b>Harm Bart</b> <i>Schur Complements and State Space Realizations</i>
12:00 – 12:30	<b>Sergey Belyi</b> <i>Realization problems in spaces with indefinite metric</i>
12:30 – 12:55	<b>Rifat Sipahi</b> <i>Cluster Treatment of Characteristic Roots for Robust Stability of Multiple Time Delayed Systems</i>
12:55 –	closing remarks

# ABSTRACTS

---

Mihaly Bakonyi  
Georgia State University  
matmmb@panther.Gsu.EDU

## *Extensions of Positive Functions on Free Groups*

Our main result states that a positive definite operator-valued function on the set of all words of length  $\leq n$  in the free group with  $m$  generators can be extended to a positive definite function on the whole group. This is rather surprising in view of the fact that a positive definite function on  $[-n, n] \times [-n, n]$  might not have a positive extension to  $\mathbf{Z}^2$ . Several related results will also be presented, including factorization of positive polynomials in noncommutative variables.

The talk is based on joint work with Dan Timotin.

---

Joseph Ball (joint work with C. Sadosky and V. Vinnikov)  
Virginia Tech  
ball@math.vt.edu

## *Multidimensional system theory, Lax-Phillips scattering and multivariable operator theory: the polydisk setting*

The one-to-one correspondence between one-dimensional linear (stationary, causal) input-state-output systems and scattering systems with one evolution operator, in which the scattering function of the scattering system coincides with the transfer function of the linear system, is well understood, and has significant applications in  $H^\infty$  control theory. Here we consider this correspondence in the  $d$ -dimensional setting in which the transfer and scattering functions are defined on the polydisk. Unlike in the one-dimensional case, the multidimensional state space realizations and the corresponding multi-evolution scattering systems are not necessarily equivalent, and the cases  $d = 2$  and  $d > 2$  differ substantially. A new proof of Andô's dilation theorem for a pair of commuting contraction operators and a new state-space realization theorem for a matrix-valued inner function on the bidisk are obtained as corollaries of the analysis.

---

**Harm Bart**  
**Erasmus University Rotterdam**  
**bart@few.eur.nl**

## *Schur Complements and State Space Realizations*

In past years, notions from system theory have been used to analyze such issues in analysis as linearization and inversion of analytic operator functions, inverse Fourier transforms, the Riemann-Hilbert boundary value problem, minimal factorization of rational matrix functions, Wiener-Hopf factorization, Wiener-Hopf and Toeplitz equations, transport theory, model reduction, Szegö limits etc. A central feature in all of this is the use of *realizations*, that is of expressions of the type

$$D + C(\lambda I - A)^{-1}B \tag{1}$$

where  $\lambda$  is a complex variable,  $A, B, C, D$  are operators (sometimes represented by matrices) and  $I$  is an appropriate identity operator. The basis for the successful application of realizations lies in the fact that (1) turns out to be ideally suited for manipulating rational matrix functions and, more generally, analytic operator functions. The main results that demonstrate this will be reviewed briefly. Further it will be pointed out that these results can be seen as "λ-versions" of certain observations on Schur complements. These observations are concerned with equivalence, extension, inversion and factorization. A central role is played by what will be called Schur coupling:  $S$  and  $T$  are said to be *Schur coupled* if for some  $2 \times 2$  operator matrix

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

involving invertible operators  $A$  and  $D$ , the operator  $S$  is the Schur complement of  $A$  in  $M$  and  $T$  is the Schur complement of  $D$  in  $M$ , i.e.,

$$S = D - CA^{-1}B, \quad T = A - BD^{-1}C.$$

In connection with this notion an open problem will be discussed too.

The talk is a report on joint work with I. Gohberg, M.A. Kaashoek and A.C.M. Ran. It also touches on research done with V.E. Tsekanovski.

---

**M. B. Bekker**  
**University of Missouri-Rolla**  
**bekkerm@umr.edu**

*Tangential Interpolation Problems for a Class of Automorphic  
Matrix-Functions (Joint work with A.A.Nudelman)*

Tangential interpolation problems are studied for matrix-valued functions of class  $(R)$  analytic in the upper half-plane with non-negative imaginary part, which are invariant under a linear fractional transformation of finite order.

---

**Tom Bella**  
**University of Connecticut**  
**bella@math.uconn.edu**

*A Björck-Pereyra-type algorithm for the Szegő-Vandermonde matrices*

In this paper we generalize the Björck-Pereyra algorithm for solving Vandermonde linear systems to polynomial Vandermonde systems  $V_P(x)$ . We show that in the case when the polynomials  $\{P_k(x)\}$  involved in  $V_P(x)$  are the *Szegő polynomials*, the properties of the corresponding *unitary Hessenberg* matrix allow us a dramatic simplification, leading to fast  $O(n^2)$  computational procedures to solve systems involving what we suggest to call Szegő-Vandermonde matrices  $V_P(x)$ .

Joint work with Yuli Eidelman, Israel Gohberg, Israel Koltracht, Vadim Olshevsky

---

**Sergey Belyi**  
**Troy University**  
**sbelyi@troy.edu**

*Realization problems in spaces with indefinite metric*

The realization problems for the Krein-Langer class  $N_\kappa$  of matrix-valued functions are being considered. We found the criterion when a given matrix-valued function from the class  $N_\kappa$  can be realized as linear-fractional transformation of the transfer function of  $\pi$ -dissipative system of the M. Livsic type with the main operator acting on a rigged Pontryagin space  $\Pi_\kappa$  (Brodskiĭ-Livsic rigged operator colligation in  $\Pi_\kappa$ ). We specify three subclasses of the class  $N_\kappa(R)$  of all realizable matrix-valued functions that correspond to different properties of a realizing system, in particular, when the domains of the main operator of a system and its  $\pi$ -adjoint coincide, when the domain of the  $\pi$ -hermitian part of a main operator is dense in  $\Pi_\kappa$ . In the case when  $\kappa = 0$  we obtain the realization results for Herglotz-Nevanlinna matrix-valued functions via conservative systems established earlier. Alternatively we show that the class  $N_\kappa(R)$  can be realized as transfer matrix-functions of some  $\pi$ -impedance systems with  $\pi$ -self-adjoint main operators in rigged Pontryagin spaces  $\Pi_\kappa$ .

This is a joint work with Yu. Arlinskii, V. Derkach, and E. Tsekanovskii.

---

Vladimir Bolotnikov  
College of William and Mary  
vladi@math.wm.edu

*Nevanlinna-Pick boundary interpolation for generalized Schur functions*

A function  $w$  is said to be in the generalized Schur class  $\mathcal{S}_\kappa$  if it is meromorphic on the unit disk  $\mathbb{D}$ , has  $\kappa$  poles in  $\mathbb{D}$  and is of the form  $w = \frac{s}{b}$  where  $s$  is a Schur function and  $b$  is a finite Blaschke product. If  $w \in \mathcal{S}_\kappa$ , then the nontangential boundary limit  $d_w(t_0) := \lim_{z \rightarrow t_0} \frac{1-|w(z)|^2}{1-|z|^2} > -\infty$  exists (finite or infinite) for every point  $t_0$  on the unit circle  $\mathbb{T}$ . If  $d_w(t_0)$  is finite, then the nontangential boundary limit  $w(t_0) := \lim_{z \rightarrow t_0} w(z)$  exists and is unimodular. This suggests the following boundary interpolation problem of the Nevanlinna-Pick type: given points  $t_i \in \mathbb{T}$ , unimodular numbers  $w_i$  and real numbers  $\gamma_i$ , find all functions  $w \in \mathcal{S}_\kappa$  such that  $d_w(t_i) = \gamma_i$  and  $w(t_i) = w_i$  for  $i = 1, \dots, n$ . The solvability criteria and description of all solutions for this and some other related problems will be presented. The talk is based on a joint work with A. Kheifets.

---

Dana Clahane  
University of California, Riverside  
dclahane@math.ucr.edu

*Norm equivalence of and composition operators between Lipschitz/Bloch spaces of the unit ball (Joint work with S. Stević)*

Let  $n$  be a positive integer, and suppose that  $\Omega$  is a domain in  $n$ -dimensional complex Euclidean space. Denote the linear space of complex-valued, holomorphic functions on  $\Omega$  by  $\mathcal{H}(\Omega)$ . If  $\mathcal{X}$  is linear subspace of  $\mathcal{H}(\Omega)$  and  $\phi : \Omega \rightarrow \Omega$  is holomorphic, then one can define the linear operator  $C_\phi : \mathcal{X} \rightarrow \mathcal{H}(\Omega)$  by  $C_\phi(f) = f \circ \phi$  for all  $f \in \mathcal{X}$ .  $C_\phi$  is called the *composition operator* induced by  $\phi$ .

A general line of investigation in this situation is to establish relationships between properties of  $C_\phi$  (for example, boundedness or compactness) to function-theoretic properties of  $\phi$ . Recently there has been interest in addressing these issues when  $\Omega$  is either the open unit polydisk or the open unit ball and  $X$  is a general  $p$ -Bloch or Lipschitz space. In a recent joint paper by the authors and Z. Zhou, the problem was solved for the polydisk. The boundedness issue for the ball remains open, though progress appears to have been recently made by Zhou jointly with R. Chen in a paper that is under submission. We will extend a classical Hardy/Littlewood characterization and norm equivalence of functions in  $p$ -Lipschitz spaces and  $(1-p)$ -Bloch spaces to the case of the unit ball. This result is of interest in and of itself, and is used in our paper to generalize to the ball a result by K. Madigan concerning

characterization of those self-maps  $\phi$  of the disk that induce bounded composition operators on  $p$ -Lipschitz spaces.

---

**Aad Dijksma**  
**University of Groningen**  
**a.dijksma@math.rug.nl**

*On the generalized Schur transform of generalized Schur and Nevanlinna functions: operator realizations, interpolation, and factorization*

The lecture is based on joint works with D. Alpay (Beer Sheva, Israel), T. Azizov (Voronezh, Russia), H. Langer (Vienna, Austria), and G. Wanjala (Mbarara, Uganda). We will discuss the generalized Schur transform of the functions mentioned in the title and some of the following topics: its effect on the operator representation of these functions, its connection with basic interpolation and boundary interpolation, and its role in the factorization of certain  $2 \times 2$   $J$ -unitary matrices into elementary factors.

---

**Michael Dritschel**  
**University of Newcastle**  
**m.a.dritschel@ncl.ac.uk**

*Interpolation on Semigroupoid Algebras*

A seminal result of Agler characterizes the so-called Schur-Agler class of functions on the polydisk in terms of a unitary colligation transfer function representation. We generalize this to the unit ball of the algebra of multipliers for a family of test functions over a broad class of semigroupoids. There is then an associated interpolation theorem. Besides leading to solutions of the familiar Nevanlinna-Pick and Carathéodory-Fejér interpolation problems and their multivariable commutative and noncommutative generalizations, this approach also allows us to consider more exotic problems. This is joint work with Stefania Marcantognini and Scott McCullough.

---

**Yuli Eidelman**  
**Tel Aviv University**  
**eideyu@post.tau.ac.il**

*The unitary completion and QR iterations for a class of structured matrices*

We consider the problem of completing a given lower triangular part  $U_L = \{u_{ij}, i \geq j\}$  to a unitary matrix. The problem of completing a given lower triangular part  $U_L$  of a block matrix to a unitary matrix  $U$  was stated and studied in the paper [1] in the assumption that the completion  $U$  admits  $LU$  and  $UL$  factorizations. We obtain the necessary and sufficient condition of existence of a unitary completion without any additional constraints and give a general formula for this completion. The talk is mainly focused on a class of structured matrices. More precisely we consider a partially specified matrix  $U = \{u_{ij}\}_{i,j=1}^N$  with the specified lower triangular part  $U_L = \{u_{ij}, i \geq j\}$  of the form

$$U_L(i+1, i) = \beta_i, \quad i = 1, \dots, N-1; \quad U_L(i, i) = d_i, \quad i = 1, \dots, N;$$

$$U_L(i, j) = p(i)q(j), \quad 1 \leq j \leq i-2, \quad 3 \leq i \leq N.$$

For such specified part the unitary completion is a structured matrix and we derive the formulas for its structure. Next we apply the unitary completion method to the solution of eigenvalue problem for a class of structured matrices via structured QR iterations. We solve the eigenvalue problem for upper Hessenberg matrices  $A$  which may be represented in the form

$$A = U + pq^*,$$

where  $U$  is a unitary matrix and  $p, q$  are given vectors. We discuss fast algorithms for solution of this problem.

This is a joint work with Dario Bini, Luca Gemignani and Israel Gohberg

## References

- [1] H. Dym and I. Gohberg, Extensions of matrix valued functions and block matrices, *Indiana University Mathematics Journal* 31: 733-765 (1982).

---

**Dario Fasino**  
**Università di Udine**  
**fasino@dimi.uniud.it**

### *Perturbative analysis of reductions to diagonal-plus-semiseparable form*

Any symmetric matrix can be brought to diagonal-plus-semiseparable form by means of the orthogonal factor appearing in the QR factorization of a suitably defined Cauchy matrix. In this reduced form, the diagonal term can be prescribed.

Owing to this characterization, we can justify the appearance of small norm submatrices in the transformed matrix, when the diagonal term is sufficiently close to the eigenvalues of the original symmetric matrix. By this analysis, we can devise an iterative algorithm

having quadratic convergence for solving the symmetric eigenvalue problem, exploiting the computational properties of diagonal-plus-semiseparable matrices.

Possibly, we will also consider the solution of the generalized tridiagonal eigenvalue problem by reducing it to a classical eigenvalue problem for diagonal-plus-semiseparable matrices.

---

**Lawrence Fialkow**  
**SUNY New Paltz**  
**fialkowl@newpaltz.edu**

### *Solution of the extremal truncated moment problem*

For a degree  $2n$  real  $d$ -dimensional multisequence  $\beta \equiv \beta^{(2n)}$   
 $= \{\beta_i\}_{i \in \mathbb{Z}_+^d, |i| \leq 2n}$  to have a *representing measure*  $\mu$ , it is necessary for the associated moment matrix  $M(n)(\beta)$  to be positive semidefinite and for the algebraic variety associated to  $\beta$ ,  $V \equiv V_\beta$ , to satisfy  $\text{rank } M(n) \leq \text{card } V$  as well as the following *consistency* condition: if a polynomial  $p(x) = \sum_{|i| \leq 2n} a_i x^i$  vanishes on  $V$ , then  $\sum_{|i| \leq 2n} a_i \beta_i = 0$ . We prove that for the *extremal* case ( $\text{rank } M(n) = \text{card } V$ ), positivity of  $M(n)$  and consistency are actually sufficient for the existence of a (unique, *rank*  $M(n)$ -atomic ) representing measure. By considering  $d = 2$  and  $n = 3$  we present the first example where consistency cannot be replaced by the weaker condition *recursiveness*. (Joint work with R.E. Curto and H.M. Moeller.)

---

**Stephan R. Garcia**  
**U.C. Santa Barbara**  
**garcias@math.ucsb.edu**

### *Complex symmetric operators (Joint work with Mihai Putinar, et. al.)*

The class of complex symmetric operators includes all normal, Hankel, and compressed Toeplitz operators (including finite Toeplitz matrices and the compressed shift). It also includes the Volterra operator, as well as many standard differential and integral operators. We discuss a concrete refinement of the polar decomposition for such operators

---

**Israel Gohberg**  
**Tel Aviv University**  
**gohberg@post.tau.ac.il**

### *Resultants for Matrix Polynomials and Applications*

The talk is based on three papers [1], [2] and [3]. In the first paper is proved the Gohberg-Heinig theorem about inversion of block Toeplitz matrices. In first and second papers is proved a version of the inverse theorem. In the second paper is presented and used a generalization of a resultant for matrix polynomials. Due to the noncommutativity of the matrix polynomials, here the resultant differs from the scalar case. There is a necessity in the general case to define the resultant not by a square but by a nonsquare matrix. This also leads to an essential difference between the presentations of the matrix and scalar inverse theorems. In the third paper (I.Gohberg and L.Lerer) is proved an other version of the matrix inverse theorem which looks like the inverse theorem for the scalar case and has important properties as in the scalar case. In the latter theorem is used a special version of the resultant of matrix polynomials which in the appearing here assymtions is defined already by a square matrix as in the scalar case. These results admit generalizations for integral operators that are continual analogs of block Toeplitz matrices. These recent results of I.Gohberg, I. Haimovici, M.A.Kaashoek and L.Lerer will be the main subject in M.A.Kaashoek's talk at this conference.

References:

[1] Gohberg I.C. and Heinig G., Inversion of finite sections of Toeplitz matrices consisting of matrices with entries from a noncommutative algebra, Rev. Roum. Math. Pures and Appl., 19 (5) (1974), 623-663 (Russian)

[2] Gohberg I. C. and Heinig G. , Resultant matrix and its generalizations. I, Resultant operator of matrix polynomials, Acta Sci. Math. 37 (1-2), (1975), 41-61 (Russian)

[3] Gohberg I. and Lerer L., Matrix generalizations of M.G. Krein theorems on orthogonal polynomials. Operator Theory: Advances and Applications 34 , (1988), Birkhauser Verlag, Basel, (137-202)

---

**Ming Gu**  
**U. C. Berkeley**  
**mgu@Math.Berkeley.EDU**

### *A Fast and Stable Polynomial Rootfinder*

Finding polynomial roots quickly and reliably has been an age-old problem in numerical analysis. While fast rootfinders are plenty, reliable ones are few. Based on semi-separable matrix techniques, we present an algorithm that finds all roots of an  $n$ -th degree polynomial in  $O(n^2)$  time and  $O(n)$  space. We also show that our method is numerically backward stable. Finally we present numerical results to demonstrate the efficiency and stability of our method.

---

**William Helton**  
**U.C. San Diego**  
**helton@math.ucsd.edu**

## *Convexity and Analysis of Noncommutative Polynomials.*

The talk concerns noncommutative polynomials and "sets" and treats questions like those an analyst would ask about ordinary polynomials.

The talk describes results mostly with Dym, McCullough, Vinnikov. and finds that positivity of second derivatives or similarly positive curvature, as well as modest deviations from positivity, produce a very rigid behavior. Convexity is very desirable for engineering purposes, but this work indicates that in systems problems whose form does not change with dimension convexity is rare. We study generalizations of convexity and find glimmerings of yet another noncommutative geometry.

If time permits the talk will also mention numerical algorithms which use noncommutative calculations to exploit structure in problems where the unknowns are matrices.

---

**Sanne ter Horst**  
**Vrije Universiteit, Amsterdam**  
**terhorst@few.vu.nl**

*All solutions to the relaxed commutant lifting problem (Joint work with  
A.E. Frazho, and M.A. Kaashoek)*

For the set of all contractive solutions to the relaxed commutant lifting problem we give a Redheffer type description involving a set of Schur class functions. For a number of special cases, including the case of the classical commutant lifting theorem, the description yields a proper parameterization of the set of all contractive solutions, but examples show that, in general, the Schur class function determining the contractive lifting does not have to be unique. Also some sufficient conditions are given guaranteeing that the corresponding relaxed commutant lifting problem has only one solution. The talk is based on [1].

## **References**

- [1] A.E. Frazho, S. ter Horst and M.A. Kaashoek, Coupling and relaxed commutant lifting, *Integral Equations and Operator theory*, to appear.
- 

**Michael Jury**  
**University of Florida**  
**jury@math.purdue.edu**

*Matrix products and interpolation in reproducing kernel Hilbert spaces*

We introduce a product operation for matrices, defined in terms of fiberwise convolution in a bundle of abelian semigroups, which generalizes the Schur (or Hadamard) product and provides a natural setting for the study of abstract Nevanlinna-Pick, Carathéodory, and mixed interpolation problems in multiplier algebras of reproducing kernel Hilbert spaces. We obtain a characterization of kernels for which a generalized NP interpolation theorem holds. This characterization contains as special cases the Carathéodory kernels of McCullough, and the NP kernels of McCullough and Quiggin. We also show that for sesquianalytic kernels satisfying some mild regularity conditions, the NP and Carathéodory kernels coincide.

---

**M.A. Kaashoek**  
**Vrije Universiteit, Amsterdam**  
**ma.kaashoek@few.vu.nl**

*The continuous analogue of the resultant for entire matrix functions*

This talk is about the operator  $R$  on  $L_1^n[-\omega, \omega]$  given by

$$(Rf)(t) = \begin{cases} f(t) + \int_{-\omega}^{\omega} d(t-s)f(s) ds, & 0 \leq t \leq \omega, \\ f(t) + \int_{-\omega}^{\omega} b(t-s)f(s) ds, & -\omega \leq t < 0. \end{cases}$$

Here  $b \in L_1^{n \times n}[-\omega, 0]$  and  $d \in L_1^{n \times n}[0, \omega]$ , and we assume that  $d(t)$  and  $b(t)$  are zero whenever  $t$  does not belong to  $[0, \omega]$  or  $[-\omega, 0]$ , respectively. This operator  $R$  is a natural continuous analogue of the classical Sylvester resultant matrix for polynomials, with the role of the polynomials being taken over by the entire  $n \times n$  matrix functions

$$\mathcal{B}(\lambda) = I_n + \int_{-\omega}^0 e^{i\lambda s} b(s) ds, \quad \mathcal{D}(\lambda) = I_n + \int_0^{\omega} e^{i\lambda s} d(s) ds.$$

The aim is to express  $\ker R$  in terms of the common spectral data of the functions  $\mathcal{B}$  and  $\mathcal{D}$ . In general, we have

$$\dim \ker R \geq \nu(\mathcal{B}, \mathcal{D}), \tag{2}$$

where  $\nu(\mathcal{B}, \mathcal{D})$  is the total multiplicity of the common eigenvalues of  $\mathcal{B}$  and  $\mathcal{D}$ . From I. Gohberg and G. Heinig, *Acta Math. Acad. Sci. Hungar* **28** (1976), 198-209, we know that for  $n = 1$  we have equality in (2), and that for  $n \geq 2$  it can happen that the inequality is strict. In this talk we consider for  $n \geq 2$  a class of pairs  $\mathcal{B}, \mathcal{D}$  for which  $\dim \ker R = \nu(\mathcal{B}, \mathcal{D})$ . The class is defined in terms of a quasi commutativity property; it contains all commuting pairs  $\mathcal{B}$  and  $\mathcal{D}$  as a strictly proper subclass. In the proof of the main theorem a related Bezout integral operator plays an important role. Applications to inverse problems will be discussed too. The talk is based on joint work with I. Gohberg, I. Haimovic and L. Lerer.

---

**M.A. Kaashoek**  
**Vrije Universteit, Amsterdam**  
**ma.kaashoek@few.vu.nl**

*The band method and the Nehari-Takagi problem*

The band method is a flexible method for solving a variety of metric constraint interpolation and extension problems. The method has evolved into increasing levels of sophistication over the past two decades. In this talk the Grassmannian version of the band method [1] is modified in order to handle the Nehari-Takagi problem. The talk is based on joint work with Orest Iftime and Amol Sasane [2].

## References

- [1] J.A. Ball, I. Gohberg, and M.A. Kaashoek. The band method and Grassmannian approach for completion and extension problems. In *Operator Theory: Advances and Applications*, Volume 87, Birkhäuser, 1996.
- [2] O. Iftime, M.A. Kaashoek, and A. Sasane, A Grassmannian band method approach to the Nehari-Takagi problem, *J. Math. Anal. Appl.*, accepted.

---

**A. Karlovich**  
**Universidade do Minho, Portugal**

*Higher order asymptotics of Toeplitz determinants with symbols in weighted Wiener algebras*

We extend a result of Böttcher and Silbermann on higher order asymptotics of determinants of block Toeplitz matrices with symbols in Wiener algebras with power weights to the case of Wiener algebras with general weights satisfying natural submultiplicativity, monotonicity, and regularity conditions.

---

**Alexander Kheifets**  
**University of Massachusetts, Lowell**  
**alexander\_kheifets@uml.edu**

*On Boundary Interpolation*

The following problem is known as untractable without additional constraints on the data  $w_0, \dots, w_n$ : find all functions  $w(z)$  analytic in the unit disk  $|z| < 1$ , such that  $|w(z)| < 1$  and

$$w(z) = w_0 + w_1(z - t_0) + \dots + w_{2n+1}(z - t_0)^{2n+1} + o((z - t_0)^{2n+1})$$

as  $z \rightarrow t_0$  nontangentially, where  $|t_0| = 1$ . Equivalence between three groups of such constraints is established: the assumption that the above asymptotics also holds for continuation by symmetry  $w(z) = \overline{w(1/\bar{z})}^{-1}$ ,  $|z| > 1$  (I. Kovalishina); Julia–Carathéodory type assumption (V. Bolotnikov and H. Dym); Hermitian property of the "Pick" matrix. The talk is based on a joint work with V. Bolotnikov.

**Martin Klaus**  
**Virginia Tech**  
**klaus@math.vt.edu**

### *Multiple eigenvalues and spectral singularities in Zakharov-Shabat systems*

We discuss the behavior of eigenvalues of nonself-adjoint Zakharov-Shabat systems as the strength of the potential is varied. Among the interesting phenomena exhibited by such systems are eigenvalue collisions and spectral singularities. However, to the best of our knowledge, previous work in this direction has been mostly numerical and limited to simple potential profiles. We will report on our recent efforts to approach such spectral problems by analytical methods for a broad class of potentials. Included are formulas for the approximate location of eigenvalue collisions and spectral singularities, and results relating the shape of the potential to collisions.

**Greg Knese**  
**Washington University in St. Louis**  
**geknese@math.wustl.edu**

### *A Schwarz Lemma on the Polydisk*

On the polydisk, there is a simple version of the infinitesimal portion of the classical Schwarz lemma: if  $f : \mathbb{D}^n \rightarrow \mathbb{D}$  is a holomorphic function, then for any  $z = (z_1, z_2, \dots, z_n) \in \mathbb{D}^n$

$$\sum_{i=1}^n (1 - |z_i|^2) \left| \frac{\partial f}{\partial z_i}(z) \right| \leq 1 - |f(z)|^2 \quad (\star)$$

The functions that satisfy equality at every point turn out to be rational inner functions that can be written as the transfer function of an  $(n+1) \times (n+1)$  symmetric unitary matrix  $U$  (i.e.  $U = U^t$  and  $U^* = U^{-1}$ ) and, in particular, belong to the Schur-Agler class on the polydisk. Moreover, we shall present some sufficient conditions for a function to satisfy equality at every point in  $(\star)$ .

---

**Israel Koltracht**  
**University of Connecticut**  
**koltracht@math.uconn.edu**

*Fast algorithms for semiseparable matrices and kernels*

Semiseparable structures is an area of active current research. I will talk about my results in this area, some of which were obtained in collaboration with I. Gohberg, T. Kailath, G. Rawitcher and others.

Semiseparable matrices and kernels are somewhat different from displacement structures which include Toeplitz, Hankel, Cauchy, Vandermonde and others, although I hear about some recent progress which may allow to include semiseparable structure in the class of displacement structures as well.

One such difference, for example is this: direct fast algorithms for matrices with displacement structure are of  $O(n^2)$ , or sometimes of  $O(n \log n)$  complexity, while semiseparable matrices admit linear complexity algorithms.

I will talk about classical semiseparable matrices and kernels. There are now extensions and generalizations, but I am not actively involved in this direction.

There is a clear duality between semiseparable matrices and kernels, for example, matrices may result from discretisation of kernels. Although it seems that the current interest is mostly in matrices, I will present parallel results for matrices and kernels, because things are sometimes more transparent in the continuous setting than in discrete.

*DEFINITIONS*

A kernel  $k(t, s)$  is called semiseparable if

$$k(t, s) = \begin{cases} a(t)b(s), & \text{if } 0 \leq s < t \leq T \\ c(t)d(s), & \text{if } 0 \leq t < s \leq T. \end{cases}$$

The corresponding Fredholm integral equation of the second kind to be solved is of the form

$$x(t) + \int_0^T k(t, r)x(r)dr = f(t), \quad 0 \leq t \leq T.$$

A matrix  $S$  is called semiseparable if for  $j, m = 0, \dots, N$

$$s_{jm} = \begin{cases} \sum_{i=1}^n a_i(j)b_i(m), & \text{if } 0 \leq j < m \leq N \\ \sum_{i=1}^n c_i(j)d_i(m), & \text{if } 0 \leq m < j \leq N \end{cases}$$

where it is assumed that  $n \ll N$ , and the equation to solve is  $Rx = f$ ,  $R = D + S$ , where  $D$  is a diagonal matrix.

For our earlier algorithms it is assumed that the operators are strongly regular. For matrices this means that all principal leading sub-matrices are non-singular, for integral operators it means that the operators

$$x(t) + \int_0^\tau k(t,r)x(r)dr = f(t), \quad 0 \leq t \leq \tau$$

are invertible for each  $0 \leq \tau \leq T$ .

In our further work we relax the strong regularity condition. For integral equations we consider certain partitioning of the interval of integration, which leads to narrow banded matrices, and similarly a certain "change of variables" for matrices leads to 3-block diagonal systems with  $n \times n$  blocks. In each case Gaussian Elimination with pivoting can be used at the expense of  $O(n)$  flops.

In our latest work we extend the latter results to semi-smooth kernels, which contain semiseparable kernels. We do not have discrete analogues for these latest results as yet.

## References

1. I. Gohberg and I. Koltracht "Numerical solution of Integral Equations, Fast Algorithms and Krein-Sobolev Equation", Numer. Math. 47, 237-288 (1985).
2. I. Gohberg, T. Kailath and I. Koltracht "Linear Complexity Algorithms for Semiseparable Matrices", IEOT, 8, 780-804 (1985).
3. R.A. Gonzales, J. Eisert, I. koltracht, M. Neumann and G. Rawitscher "Integral Equation Method for the Continuous Spectrum Radial Schroedinger Equation", J. Comp. Physics 134, 134-149 (1997).
4. I. Koltracht "Linear Complexity Algorithm for Semiseparable Matrices, IEOT 29, 313-319 (1997).
5. S.-Y. Kang, I. Koltracht and G. Rawitscher "Nystrom-Clenshaw-Curtis Quadrature for Integral Equations with Discontinuous Kernels", Math. of Comp. 72, No. 242, 729-756 (2002).

---

**Ilya Krishtal**  
**Washington University in St. Louis**  
**krishtal@math.wustl.edu**

*Linear Relations and Operator Semigroups (Joint work with A.G. Baskakov)*

Let  $X$  be a complex Banach space,  $EndX$  be a Banach algebra of endomorphisms of the space  $X$ , and  $T : \mathbb{R}_+ = (0, \infty) \rightarrow EndX$  be a strongly continuous semigroup with

the infinitesimal generator  $A_0 : D(A_0) \subseteq X \rightarrow X$ . We also consider the linear relation (linear subspace)  $\mathbf{A} \subseteq X \times X$  which is defined as follows. A pair  $(x, y)$  belongs to  $\mathbf{A}$  if  $x \in \overline{ImT} = \cup_{t \geq 0} ImT(t)$  and

$$T(t)x - T(s)x = \int_s^t T(\tau)y d\tau, \quad 0 < s \leq t < \infty.$$

It is easily seen that  $\mathbf{A}$  is a closed subspace and that  $A_0 \subseteq \mathbf{A}$  if we identify the linear operator with its graph. Any linear relation  $\mathbf{A}$  which commutes with operators  $T(t)$ ,  $t \geq 0$ , and satisfies  $A_0 \subseteq \mathbf{A} \subseteq \mathbf{A}$  is called a *generator* of the semigroup  $T$ . A generator is called *basic* if its resolvent  $\rho(\mathbf{A})$  contains the half-plane  $\mathcal{C}_w = \{\lambda \in \mathcal{C} : Re\lambda > w\}$ , where  $w$  is the type of the semigroup  $T$ .

We obtain necessary and sufficient conditions for the existence and uniqueness of the basic generator of strongly continuous and other types of operator semigroups as well as revisit certain classical results.

**Michael Lauzon**  
**Brown University**  
**mlauzon@math.brown.edu**

*Scalar and vector Muckenhoupt weights (Joint work with S.Treil)*

Given a weight  $w : \mathbb{R} \mapsto [0, \infty)$ , we define  $L^2(w)$  by

$$\|f\|_{L^2(w)}^2 = \int |f(t)|^2 w(t) dt.$$

It is well known that the Hilbert transform is bounded on  $L^2(w)$  if and only if  $w$  satisfies the  $\mathcal{A}_2$  condition:

$$\frac{1}{|I|} \int_I w(t) dt \frac{1}{|I|} \int \frac{1}{w(t)} dt$$

for all intervals  $I$ . Similar statements are true for vector valued functions, where you consider a positive-matrix valued function  $W$ , and define  $L^2(W)$  by

$$\|f\|_{L^2(W)}^2 = \int (W(t)f(t), f(t)) dt.$$

A more recent result, by Nazarov, Treil et. al. is that the Hilbert transform is bounded if and only if  $W$  satisfies the  $\mathcal{A}_{2,2}$  condition:

$$\left\| \left( \frac{1}{|I|} \int_I W \right)^{1/2} \left( \frac{1}{|I|} \int_I W^{-1} \right)^{1/2} \right\| \leq C$$

The focus of this talk will be the relationship between the scalar  $\mathcal{A}_2$  and the vector  $\mathcal{A}_{2,2}$  condition. For example, if we consider an  $\mathcal{A}_{2,2}$  weight  $W$  and a fixed nonzero vector  $x$ , the scalar weight formed by setting  $w(t) = (W(t)x, x)$  will be a scalar  $\mathcal{A}_2$  weight. What is nontrivial is showing that it is possible for a matrix weight  $W$  to be uniformly  $\mathcal{A}_2$  (in the sense that the  $(W(t)x, x)$  are  $\mathcal{A}_2$  with a uniform constant in  $x$ ) but not  $\mathcal{A}_{2,2}$ . Such a weight will be constructed in this talk, and a space-filling curve will play an interesting roll in this construction.

---

**Yuri Lyubich**  
**Technion**  
**lyubich@tx.technion.ac.il**

*The Sylvester-Ramanujan equations and related topics*

A classical system of algebraic equations is treated as a finite power moment problem in the complex plane and investigated on this base. Being originated from the algebraic theory of binary forms, this system is closely connected with an extraordinary number of different subjects in the classical and modern analysis (Gauss type quadrature formulas, continuous fractions, Pade approximation, Hankel matrices, orthogonal polynomials, etc.). A survey of these relations is presented.

---

**Michael Neumann**  
**University of Connecticut**  
**neumann@math.uconn.edu**

*Reduced Rank Nonnegative Matrix Factorization for Symmetric  
Nonnegative Matrices (Joint work with M. Catral, L. Han, and R. J.  
Plemmons)*

Let  $V \in \mathbb{R}^{m,n}$  be a nonnegative matrix. The nonnegative matrix factorization (NNMF) problem consists of finding nonnegative matrix factors  $W \in \mathbb{R}^{m,r}$  and  $H \in \mathbb{R}^{r,n}$  such that  $V \approx WH$ . Lee and Seung proposed an algorithm which finds nonnegative  $W$  and  $H$  such that  $\|V - WH\|_F$  is minimized. After examining the case in which  $r = 1$ , we consider the case in which  $m = n$  and  $V$  is symmetric. We focus on questions concerning when the best approximate factorization results in the product  $WH$  **being symmetric** and on cases in which the best approximation **cannot be a symmetric matrix**. We show that the class of positive semidefinite symmetric nonnegative matrices  $V$  generated via a Soules basis, admit for every  $1 \leq r \leq \text{rank}(V)$ , a nonnegative factorization  $WH$  which coincides with the best

approximation in the Frobenius norm to  $V \in \mathbb{R}^{n,n}$  of rank not exceeding  $r$ . We will also suggest applications of our findings to the  $cprank$  problem for completely positive matrices.

---

**Vadim Olshevsky**  
**University of Connecticut**  
**olshevsky@math.uconn.edu**

*Eigenvalue problems and quasiseparable matrices*

Tridiagonal matrices are in the heart of several algorithms for solving a general eigenvalue problem. In this talk we generalize some of the algorithms replacing a tridiagonal matrices by the more general quasi-separable matrices. The latter are defined as matrices for which all  $2 \times 2$  square partitions yield low rank (2,1) and (1,2) blocks. The latter class of matrices includes not only tridiagonal but also unitary Hessenberg matrices that have been a subject of a number of publications in the last decade.

We show how every symmetric matrix can be reduced to a quasi-separable matrix. We further make use of the observation that the quasiseparable structure is inherited under QR iterations. We provide efficient algorithms to compute the QR factorization for a quasiseparable  $R$ , and to further multiply them in the reversed order  $RQ$ . The proposed family of quasiseparable QR algorithms includes the classical tridiagonal QR algorithm as a special case.

In the rest of the talk we show that the characteristic polynomials associated with the leading blocks of a quasiseparable matrix satisfy three-term and two-term recurrence relations. Moreover, the class of all such polynomials includes the classical orthogonal polynomials (tridiagonal case) and the Szegő polynomials (unitary Hessenberg case) as special cases. This observation leads to generalizations of the Sturm sequences method as well as to a generalization of the Newton method.

Joint work with Yuli Eidelman and Israel Gohberg

---

**David Opela**  
**Washington University in Saint Louis**  
**opela@math.wustl.edu**

*Extension and Dilation Theorems for Operators Satisfying Relations*

Andô's dilation theorem states that any pair of commuting Hilbert space contractions can be dilated (or extended) to a pair of commuting unitaries (co-isometries, resp.). We generalize this theorem by proving that one can dilate pairs of contractions to pairs of

unitaries while preserving ‘relations’ different from  $AB = BA$  (commutation), for example the relation  $A^n B = B A^k$ . It is well-known that Andô’s theorem does not generalize to triples of commuting contractions. Therefore it is interesting to study the possibility of dilating  $n$ -tuples of contractions satisfying some relations. We generalize Andô’s theorem in this direction, too.

---

**Z. Pan**  
**Saginaw Valley State University**  
**pan@svsu.edu**

*Separating Operators by Linear Functionals*

Let  $L(X)$  be the set of all bounded linear operators on a Banach space  $X$ . Given a set of linear functionals  $\mathcal{T}$  on  $L(X)$  and a subset  $\mathcal{B}$  of  $L(X)$ , it is natural to ask: does  $\mathcal{T}$  separate every operator outside  $\mathcal{B}$  from  $\mathcal{B}$ ? Various results, some well-known and some definitely not, of this nature will be discussed.

---

**Artem Pulemyotov**  
**Cornell University**  
**artem@math.cornell.edu**

*Image of a Jacobi field (Joint work with Yu.M. Berezansky)*

The concept of a Jacobi field is, in a sense, an infinite-dimensional analogue of the concept of a Jacobi matrix. By definition, a Jacobi field  $J$  is a family of three-diagonal matrices with operator entries. These matrices are indexed by the vectors of a Hilbert space  $H_+$  imbedded densely and continuously into another Hilbert space  $H$ . Each matrix is required to define an operator in the Fock space  $\mathcal{F}(H)$ . Among other requirements, these operators must be selfadjoint and commuting. We remark that the theory of Jacobi fields relates to white noise calculus, stochastic processes, non-Abelian integrable systems, and polynomials in multiple variables.

The spectral measure  $\rho$  of the Jacobi field  $J$  is defined on the Hilbert space  $H_-$ , the dual of  $H_+$ . The talk aims to study the image  $\rho_K$  of the measure  $\rho$  under a linear operator  $K^+$ . We construct a family of operators whose spectral measure equals  $\rho_K$  and develop the spectral theory of this family. We obtain an analogue of the Wiener-Itô decomposition for  $\rho_K$ . It is worth noting that, for a certain choice of  $J$  and  $K^+$ , the measure  $\rho_K$  appears to be a (fractional) Lévy noise measure.

---

**Lothar Reichel**  
**Kent State University**  
**reichel@math.kent.edu**

*Tikhonov Regularization of Large-Scale Problems*

The solution of large-scale ill-posed problems has recently received considerable attention. Many of the available solution methods are based on Tikhonov regularization. This talk surveys iterative methods for Tikhonov regularization. Several of these methods explore the connection between orthogonal polynomials, Gauss quadrature, and Lanczos bidiagonalization. Methods for unconstrained and constrained regularization problems will be discussed.

---

**Leiba Rodman**  
**College of William and Mary**  
**lxrodm@MATH.WM.EDU**

*Convex Invertible Sets of Matrices*

A set  $S$  of matrices is said to be *convex invertible* if it is convex, every element in  $S$  is invertible, and  $S$  is closed under inversion. Recent results on the structure of certain classes of convex invertible sets will be reviewed. Open problems in this area will be stated.

One result describes the possible convex invertible sets of the form  $A \circ X$ , where  $A$  is a fixed complex matrix,  $X$  is an arbitrary matrix with positive entries, and  $\circ$  stands for the entrywise, a. k. a. Hadamard, multiplication.

Another result describes a construction of convex invertible sets in the commutative subalgebras of the algebra of real  $2 \times 2$  matrices.

The talk is based on two papers; one joint with C.-K. Li, the other joint with I. Lewkowicz and E. J. Yarkoni.

---

**Joachim Rosenthal**  
**University of Zürich**  
**rosenthal@math.unizh.ch**

*Building Public Key Crypto-Systems from Operators over Semi-Rings*

Cryptography has a long history and its main objective is the transmission of data between two parties in a way which guarantees the privacy of the information. There are other

interesting applications such as digital signatures, the problem of authentication and the concept of digital cash to name a few. The proliferation of computer networks resulted in a large demand for cryptography from the private sector.

A basic building block in public key cryptography are the one-way trapdoor functions. These are one-one functions which can be efficiently computed. The inverse function can however only be computed if some additional trapdoor is known.

In this talk we explain some new ideas on how to build one-way trapdoor functions from semi-groups over finite simple semi-rings and their actions on finite semi-modules. The presented results constitute joint work with Gerard Maze and Chris Monico.

---

**James Rovnyak**  
**University of Virginia**  
**rovnyak@virginia.edu**

*Inverse problems for canonical differential equations with singularities*  
(Joint work with L.Sakhnovich)

A canonical differential equation is a system of the form

$$\frac{dY}{dx} = izJH(x)Y, \quad 0 \leq x < \ell,$$

subject to suitable initial conditions and satisfying  $H(x) \geq 0$ . Here  $H(x)$  has  $2m \times 2m$  matrix values,  $Y(x, z)$  has  $2m \times 1$  matrix values, and  $J$  is a signature matrix. There is an extensive literature on such systems. In most cases, the Hamiltonian  $H(x)$  is assumed to be regular, that is, locally integrable on  $[0, \ell)$ . This lecture is a preliminary report on joint work with L. A. Sakhnovich in which we allow  $H(x)$  to have a finite or countably infinite number of singularities. More specifically, the purpose of the lecture is to describe classes of inverse problems in which the solution by means of operator identities produces systems such that  $H(x) \geq 0$  except at certain isolated singular points. To formulate such problems, the usual notion of a spectral function is replaced by a more general notion of spectral data, which is derived from the Kreĭn-Langer integral representation of a generalized Nevanlinna function. A number of examples will be given in which the calculations can be explicitly carried through.

---

**Lev Sakhnovich**  
**lev.sakhnovich@verizon.net**

*Integrable operators*

---

**Rifat Sipahi**  
**University of Connecticut**  
**rifat@engr.uconn.edu**

*Cluster Treatment of Characteristic Roots for Robust Stability of Multiple Time Delayed Systems (Joint work with N.Olgac)*

A unique perspective is presented for studying the stability robustness of systems with multiple delays. The general class is given in the form of  $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \sum_{j=1}^p \mathbf{B}_j\mathbf{x}(t - \tau_j)$  where  $\mathbf{x} \in \mathfrak{R}^n$ ,  $\mathbf{A}$  and  $\mathbf{B}_j \in \mathfrak{R}^{n \times n}$  and the positive time delays  $\tau_1, \tau_2, \dots, \tau_p$  are rationally independent from each other. The robust stability assessment of this dynamics against delay uncertainties is an open problem. In our pursuit, two unrecognized propositions surfaced, which, in turn, gave rise to a unique methodology, Cluster Treatment of Characteristic Roots (CTCR). *Proposition I* claims that the entire set of potential stability switching loci in  $\boldsymbol{\tau}$  space can be reduced to a *manageably small number of*, so called, *kernel hyperplanes*. The others in the set, are called 'offspring hyperplanes'. *Proposition II* shows that as we move parallel to any delay axis crossing the kernel hyperplane and its respective offspring, the tendency of the stability switching is invariant (either stabilizing or destabilizing). An interesting holographic mapping is deployed to determine the *kernel* and *offspring* hyperplanes exhaustively. They partition the  $p$ -dimensional  $\boldsymbol{\tau}$  domain into regions where the stability posture remains unchanged (i.e., either stable or unstable). The new method, CTCR, identifies such regions with a constant number of unstable roots ( $NU$ ). Obviously  $NU = 0$  implies stable regions. We also demonstrate the applicability of CTCR on real life examples, such as variable pitch milling and trajectory tracking.

*Remark: This problem is claimed to be in NP-Hard class (Toker and Ozbay, 1996). CTCR attempts to bring an efficient algorithm to contrast this claim. So far, we demonstrate the ability in 2 and 3 delay cases, which are unprecedented in the literature.*

---

**Ilya Spitkovsky**  
**College of William and Mary**  
**ilya@MATH.WM.EDU**

*Factorization in Wiener algebras on ordered abelian groups*

This is a survey of the current state for the factorization theory of matrix functions defined on connected compact additive abelian groups. The related questions from the theory of Toeplitz operators will also be discussed. —————

---

**Saida Sultanic**  
**Washington University in St. Louis**  
**sultanic@math.wustl.edu**

*Commutant Lifting Theorem for the Bergman Space*

The question of finding the right analogue of the Commutant Lifting Theorem for the Bergman space  $L_a^2(\mathbb{D})$  will be discussed. We will present our work on this problem and the analogous problem for the weighted Bergman spaces  $L_{a,\alpha}^2(\mathbb{D}, dA)$ ,  $-1 < \alpha < \infty$ .

---

**Tavan T. Trent**  
**The University of Alabama**  
**ttrent@gp.as.ua.edu**

*A Vector-valued Corona Type Theorem on the Polydisk*

Let  $\{f_j\}_{j=1}^\infty \subset H^\infty(D^n)$ . Assume that  $\epsilon^2 \leq \sum_{j=1}^\infty |f_j(z)|^2 \leq 1$  for all  $z \in D^n$ . We find analytic functions on  $D^n$ ,  $\{u_j\}_{j=1}^\infty$ , belonging to the Orlicz-type space,  $\exp(L^{\frac{1}{2n-1}})$  and satisfying  $\sum_{j=1}^\infty f_j u_j \equiv 1$  in  $D^n$ . Note that  $H^\infty(D^n) \not\subseteq \text{BMO}(D^n) \not\subseteq \exp(L^{\frac{1}{2n-1}}) \not\subseteq \bigcap_1^\infty H^p(D^n)$ .

---

**Eduard Tsekanovskii**  
**Niagara University**  
**tsekanov@niagara.edu**

*A general realization theorem for Herglotz-Nevalinna matrix-valued functions*

New special types of stationary conservative impedance and scattering systems, the so-called non-canonical systems, involving triplets of Hilbert spaces and projection operators, are considered. It is established that any matrix-valued Herglotz-Nevalinna function of the form

$$V(z) = Q + Lz + \int_{\mathbf{R}} \left( \frac{1}{t-z} - \frac{t}{1+t^2} \right) d\Sigma(t)$$

can be realized as a transfer function of such a new type of conservative impedance system. In this case we show that the realization can be chosen such that the main and projection operators of the realizing system satisfy a certain commutativity condition if and only if  $L = 0$ . It is also shown that  $V(z)$  with an additional condition (namely,  $L$  is invertible or  $L = 0$ ), can be realized as a linear fractional transformation of the transfer function of a non-canonical scattering system. In particular, this means that any scalar Herglotz-Nevalinna function can be realized in the above sense. We discuss also conditions on  $V(z)$  when the minimal realization is in the terms of canonical generalized systems of the Livšic type (Brodskiĭ-Livšic rigged operator colligations). The established general realization theorems give a solution of a problem (scalar case and partly matrix-valued case) from “Unsolved problems in Mathematical Systems and Control Theory”, Princeton University Press, 2004 and are strongly connected with, and complement the recent results by J. Ball and O. Staffans.

This talk is based on joint work with S. Belyi, S. Hassi, and H. de Snoo.

---

**Eugene Tyrtysnikov**  
**Institute of Numerical Mathematics, Russian Academy of Sciences**  
**tee@bach.inm.ras.ru**

### *Separability concepts and multilevel matrices*

Separability in multivariate functions means separation of variables; for matrices viewed as functions of row and column indices it means separation of these indices (dyadic or skeleton decompositions). Above all, separability is a powerful approximation tool facilitating many constructions in analysis and numerical methods. However, separability properties depend on domains where the variables belong; when speaking of matrices, one has to take into account that separability (low-rank approximations) should be studied for suitably chosen submatrices or even for some subdomains of the entries. The former leads eventually to multilevel block matrices with low-rank blocks or certain tensor properties [2], the latter is in relation with semi and quasi separable matrices, and more generally, recently introduced piecewise separable matrices [5]. Moreover, separability features large in the analysis and construction of preconditioners in iterative processes for structured (Toeplitz etc.) matrices; in this role, it underlines recent positive and negative results on “super-linear” preconditioners and even may pave a way to “best possible” preconditioners [4].

The goal of this talk is a revision of interrelations between various manifestations of the separability. From the viewpoint of computations, separability provides a tremendous reduction of the data-representation parameters. Since the number of these parameters tends to increase during matrix operations, a simple and natural idea is coming of a suitable truncation at each step. Such an idea was successfully applied to computation of the inverse for low-displacement-rank matrices (V.Pan et al; D.Bini et al) and then for matrices of low tensor rank in the case of two factors [3]. Recently it has been comprehended that the

constructions used therein are particular cases of a unified general framework that applies not only to the Newton-Schultz method for the inverse matrices, but also to other iterative processes for computation of many other functions of matrices (e.g. sign, square root, etc.) [1].

#### REFERENCES

[1] W. Hackbusch, B. Khoromskij and E. Tyrtyshnikov, Approximate iterations for structured matrices, Max-Planck-Institut für Mathematik in den Naturwissenschaften, Leipzig, 2005.

[2] V. Olshevsky, I. Oseledets, E. Tyrtyshnikov, Tensor properties of multilevel Toeplitz and related matrices, Linear Algebra Appl., accepted for publication (2005).

[3] I. Oseledets and E. Tyrtyshnikov, Approximate inversion of matrices in the process of solving a hypersingular integral equation, Comp. Math. and Math. Phys., Vol. 45, No. 2 (2005), 302–313 (translated from "JVM i MFi", Vol. 45, No. 2 (2005), 315–326).

[4] I. Oseledets and E. Tyrtyshnikov, Best possible circulant preconditioners: a unifying approach. Presentation at the International Conference on Matrix Methods and Operator Equations, June 22-25, Moscow.

[5] E. Tyrtyshnikov, Piecewise Separable Matrices, submitted to Calcolo, 2005.

---

**Marc Van Barel**  
**Katholieke Universiteit Leuven**  
**marc.vanbarel@cs.kuleuven.ac.be**

*The Lanczos reduction to semiseparable matrices and applications in computing dominant subspaces (Joint work with Nicola Mastronardi, Mieke Schuermans, Sabine Van Huffel, Raf Vandebril)*

The Lanczos tridiagonalization is the classical way to compute an approximation of dominant singular subspaces [1, 2, 3, 4, 5]. Recently, a reduction of matrices in semiseparable form has been introduced [6]. The latter reduction has the same convergence properties of Lanczos tridiagonalization for the "extremal" eigenvalues and it is based on Householder and Givens transformations.

The interesting feature of the latter algorithm is that, if gaps are present in the spectrum, after few steps the transformed matrix has almost a diagonal block structure, and the largest eigenvalues of the left-top block converge to the largest eigenvalues of the original matrix.

In this talk we consider a new reduction of a matrix in semiseparable form. Similarly to the Lanczos tridiagonalization, the main computation of each step of the the new algorithm is a product of the original matrix by a vector. This can be done in an efficient way exploiting the structure of the original matrix (i.e. sparsity, structure, ...). Moreover, during the reduction the already partially computed semiseparable submatrix is almost block diagonal, and the largest eigenvalues of the left-top block converge to the largest eigenvalues of the original matrix.

To show the efficiency of the new algorithm, as an application, a time–domain estimation method to extract the parameters characterizing signals from the magnetic resonance spectroscopy is considered. In the latter application, the most expensive part is the computation of the singular subspace associated to the largest singular values of a Hankel matrix of large size.

## References

- [1] D. Calvetti, L. Reichel, and D. Sorensen. An implicitly restarted Lanczos method for large symmetric eigenvalue problems. *Electronic Transactions on Numerical Analysis*, 2:1-21, 1994.
- [2] J. Cullum and R. A. Willoughby. *Lanczos Algorithms for Large Symmetric Eigenvalue Computations: Theory*, volume 3 of *Progress in Scientific Computing*. Birkhauser, Boston, 1985.
- [3] R.D. Fierro. Lanczos, Householder transformations, and deflation for fast and reliable subspace computation. *Numerical Linear Algebra with Applications*, 8:245-264, 2001.
- [4] G. H. Golub and C. F. Van Loan. *Matrix Computations*. The Johns Hopkins University Press, 1996, Third edition.
- [5] B. N. Parlett. *The symmetric eigenvalue problem*. Classics in Applied Mathematics. SIAM, Philadelphia, PA, 1998.
- [6] R. Vandebril, M. Van Barel and N. Mastronardi. An orthogonal similarity reduction of a matrix into semiseparable form. TW355, Dept. Computer Science, K.U.Leuven, 2003, accepted for publication in *SIAM J. Matrix Anal. Appl.*
- [7] R. Vandebril, M. Van Barel and N. Mastronardi. An implicit  $QR$ -algorithm for semiseparable matrices to compute the eigendecomposition of symmetric matrices. TW367, Dept. Computer Science, K.U.Leuven, 2003.
- [8] R. Vandebril, M. Van Barel and N. Mastronardi, N. A  $QR$ -method for computing the singular values via semiseparable matrices, *Numerische Mathematik*, to appear.

---

**Paul Van Dooren**  
**Univervsite catholique de Louvain**  
**vdooren@csam.ucl.ac.be**

*On computing the complex passivity radius (Joint work with Michael L. Overton)*

We characterize the complex passivity radius of a rational transfer matrix  $G(s) := C(sI_n - A)^{-1}B + D$  and propose an approach to compute it. The method depends on computing the smallest structured indefinite perturbation to a Hermitian matrix that makes it singular. We consider both additive and multiplicative perturbations, giving details for the additive case. In both cases, the smallest indefinite perturbation can be efficiently computed by solving a unimodal optimization problem in a real parameter. The passivity radius can be computed by minimizing the smallest singularity-inducing multiplicative indefinite perturbation of a frequency-dependent matrix over the imaginary axis.

---

**Raf Vandebril**  
**Katholieke Universiteit Leuven**  
**raf.vanderbril@cs.kuleuven.ac.be**

*A Levinson-like solver for semiseparable plus band matrices*

In this talk we will derive a solver for systems of equations with higher order generator representable semiseparable plus band matrices. The solver we will derive is based on the Levinson algorithm, which is used for solving strongly nonsingular Toeplitz systems.

The solver is constructed in a similar way as the solver for Toeplitz systems: firstly a Yule-Walker-like equation needs to be solved, and secondly this solution is used for solving a linear equation with an arbitrary right-hand side.

The overall complexity of the presented solver is  $6(l+p)^2n$  plus lower order terms, where  $l$  stands for the bandwidth of the band matrix, and  $p$  stands for the semiseparability rank of the semiseparable matrix.

In a final part numerical experiments are performed. Attention is paid to the timing and the accuracy of the described methods.

---

**Hugo Woerdeman**  
**Drexel University**  
**hjlw27@drexel.edu**

*The Operator Valued Autoregressive Filter Problem and the Suboptimal Nehari Problem in Two Variables*

The two-variable autoregressive filter problem for operator valued functions is considered. Its solution introduces a commutativity condition on operator matrices constructed from the two-variable operator valued Toeplitz matrix. The subsequent construction may be viewed as a two-variable analog of Silverman's algorithm.

In addition, the two variable operator valued Nehari problem is considered. Recently Ferguson and Lacey showed that a bounded little Hankel operator has a bounded symbol. In general, however, the infinity norm of the symbol is larger than the operator norm of the little Hankel operator, distinguishing it from the classical Nehari result. In this presentation we shall present sufficient conditions for when the norm of the symbol satisfies the same bound as the norm of the little Hankel operator.

This presentation is based on joint work with Jeffrey Geronimo.

---