Math2410 Quiz #1

Question 1 (10 points total)
Solve the first-order differential equation
\[ \frac{dy}{dt} = \frac{t}{t^2 y + y}. \]

Solution:
To solve this, we separate variables and integrate:
\[ \frac{dy}{dt} = \frac{1}{y} \frac{t}{1 + t^2} \]
i.e.
\[ y \frac{dy}{dt} = \frac{t}{1 + t^2} \]
so
\[ \int y \, dy = \int \frac{t}{1 + t^2} \, dt \]
and so
\[ \frac{y^2}{2} = \frac{1}{2} \ln(1 + t^2) + c \]
i.e.
\[ y = \pm \sqrt{\ln(1 + t^2) + k} \]
where \( k = 2c \).

Note: We haven’t missed any solutions here, since there is no equilibrium solution that makes \( \frac{1}{y} \) equal to 0.

Some common mistakes:

1. You should, when possible, solve for \( y \) (not just leave the solution implicitly defined).
2. Remember that, in general, \( \sqrt{a + b} \neq \sqrt{a} + \sqrt{b} \) so \( \sqrt{\ln(1 + t^2) + k} \neq \sqrt{\ln(1 + t^2)} + k \).
3. Forgetting to write \( \pm \) may seem like a minor error, but it means that you miss out on many solutions. If you say \( y = \sqrt{\ln(1 + t^2) + k} \) then you can’t solve the IVP with initial condition \( y(0) = -1 \) (or any other negative number). So, you’re missing half of the solutions. Remember that the general solution should deal with all initial conditions.
4. The biggest mistake to make here is forgetting to revise your integration. Now (early in the semester) is the time to do this.