

Lane-Emden problem : survey about symmetries.

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During this talk, we will be interested in the problem

$$\begin{cases} -\Delta u = |u|^{p-2}u, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

where Ω is open bounded in \mathbb{R}^N , $N \geq 2$ and $2 < p < \frac{2N}{N-2}$. More exactly, we will study symmetries of ground states and least energy nodal solutions (lens). Respectively, they are one-signed solutions and nodal solutions with minimum energy

$$\mathcal{E}_p : H_0^1(\Omega) \rightarrow \mathbb{R} : u \mapsto \frac{1}{2} \int_{\Omega} |\nabla u|^2 - \frac{1}{p} \int_{\Omega} |u|^p.$$

In '73, B. Gidas, W. M. Ni and L. Nirenberg proved, by using the “moving planes” method, that ground states respect the symmetries of Ω , when Ω is “convex”. For not-convex domains and lens, this technic fails. During the talk, we will see that, on rectangles, lens respect symmetries of the second eigenfunction of Δ with Dirichlet boundary conditions, at least for p close to 2. On radial domains, lens is even with respect to $N - 1$ independant directions and odd with respect to the orthogonal one, for p close to 2.

In general, ground state (resp. lens) respect symmetries of its projection on first (resp. second) eigenspace of Δ with DBC, for p close to 2. Moreover, accumulation points of solutions when $p \rightarrow 2$, in some sense, minimize a limit energy functional on a limit manifold. It will be illustrated on the square. It implies, for large p , some symmetry breaking for lens on rectangles. But what about others domains and p large ?

To finish, we will discuss what happens about symmetries if we are working with Neumann boundary conditions, with q -Laplacian or if we are changing the non-linearity in the equation. We will spend some time for several open problems. Results will be illustrated by using the Mountain-Pass algorithm of Y. S. Choi and P. J. McKenna.