

1. Consider the partially decoupled system

$$\begin{cases} \frac{dx}{dt} = xy - 1, \\ \frac{dy}{dt} = y + 1 \end{cases}$$

- a) Find the equilibrium solutions of the system;
  - b) Derive the general solution;
  - c) Find the solution that satisfies  $x(0) = 1, y(0) = 0$ ;
2. Regarding the system of equation in Problem 1 and using Euler method with  $\Delta t = 0.5$ , find the numerical solution with initial data  $x(0) = 1, y(0) = 0$  for  $0 \leq t \leq 2$ . Draw the solution curve in the phase plane using the numerical results above.
3. Consider the system of equation

$$\frac{dY}{dt} = AY,$$

where

$$A = \begin{pmatrix} -2 & -1 \\ 1 & 4 \end{pmatrix}$$

- a) Find the straight line solutions of the system of equation;
  - b) Find the general solution;
  - c) Find the solutions that satisfies  $Y(0) = (0, -1)$ .
  - d) Determine the type of equilibrium point at the origin.
  - e) Sketch the phase portrait for the linear system.
4. Using

$$A = \begin{pmatrix} 1 & 4 \\ -1 & 1 \end{pmatrix}$$

answer the questions in Problem 3.

5. Consider the following second order equation for a damped harmonic oscillator

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0.$$

- a) Convert this equation into a first order linear system;
- b) Compute the eigenvalues;
- c) Find the general solution.