

PROBABILISTIC METHODS IN GEOMETRY AND ANALYSIS

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ABSTRACT. These are topics we covered.

April 20, 2017

Introduction: motivation and history. Spectral properties, geometry and probability.

April 24, 2017Markov semigroups: basic definitions including different types of continuity, invariant measure, Gaussian on \mathbb{R}^n , Markov semigroups and Markov processes.**April 27, 2017**

Markov semigroups and Markov processes, Ornstein-Uhlenbeck semigroup, infinitesimal generators.

May 1, 2017

No class: Tag der Arbeit.

May 4, 2017Infinitesimal generators on $L^2(X, \mu)$, where μ is an invariant measure, examples, Hille-Yosida theorem for contraction C_0 -semigroups, more properties of semigroups and their generators.**May 8, 2017**

Closable operators, carré du champ operators, examples, properties, symmetric semigroups, diffusion operators

May 11, 2017

Chain Rule, examples (Gaussian, OU, Laplace-Beltrami operator and Bochner-Weitzenböck formula).

May 15, 2017

Date: July 6, 2017 .

Laplace-Beltrami operator for the sphere \mathbb{S}^2 curvature-dimension inequality, examples, gradient estimates: semigroup formulation for $CD(0, \infty)$, Poincaré inequalities (local and classical, reverse), spectral gap.

May 18, 2017

Proof of the PI, Logarithmic Sobolev inequalities (Sobolev inequalities, curvature-dimension inequality and the log Sobolev inequality).

May 22, 2017

Strong gradient estimate, reformulation in terms of the entropy and Fisher information, the Dirichlet form, LSI implies PI.

May 25, 2017

No class: Ascension Day.

May 29, 2017

Ergodicity, ergodicity and invariant measures, log Sobolev inequalities: a short history, LSI and E.Nelson's hypercontractivity.

June 1, 2017 LSI and E.Nelson's hypercontractivity, Jacobi's trick (ground state transformation) and hypercontractivity.

June 5, 2017

No class: Pentecost Break.

June 8, 2017

No class: Pentecost Break.

June 12, 2017

Dirichlet forms, invariant and reversible measures, Markov processes' interpretation, the domain of a Dirichlet form.

June 15, 2017

No class: Corpus Christi.

June 19, 2017

Intrinsic distance for a carré du champ operator, examples, connection to Rademacher's theorem, sub-Riemannian manifolds, Hörmander's or bracket-generating condition, Carnot-Carathéodory metric.

June 22, 2017

Metric spaces: length metric spaces, geodesics, geodesic spaces, Hopf-Rinow–Cohn-Vossen theorem, Chow–Rashevskii theorem for sub-Riemannian manifolds, horizontal gradients, differences from the Riemannian case, hypoelliptic operators and Hörmander's theorem, semi-elliptic, elliptic, uniformly elliptic operators.

June 26, 2017

Heisenberg group as an example of a sub-Riemannian manifold, generalized curvature-dimension inequality, gradient estimates, heat kernel estimates.

June 29, 2017

On-diagonal heat kernel estimates. Couplings of probability measures, couplings of stochastic processes, coupling time, Aldous inequality, successful couplings, maximal couplings.

July 3, 2017

Markovian couplings, Lindvall and Rogers reflection (mirror) coupling for diffusions, application of the Lindvall-Rogers coupling to gradient estimates of harmonic functions.

July 6, 2017

Maximal Markovian couplings in \mathbb{R}^d , synchronous couplings and gradient estimates for the heat semigroup.

July 10, 2017

No class.

July 13, 2017

No class.