

MATH 2210Q
Spring 2009
Sarah Glaz

Final Exam Guidelines: Material and Review Suggestions

Date and place: Saturday, May 9, 1:00 – 3:00 pm, MSB 311

Additional office hours before exam: Friday, May 8, 1:30 – 2:30, and Saturday, May 9, 12:00 – 1:00

My Policy: This is a one hour exam, but all students may stay for as long as they need to finish the exam.

University Policy: Please note that the University of Connecticut requires students to obtain permission in writing from the Dean of Students in order to take a make-up Final Exam. If you have a conflict with the Final Exam schedule, talk to me as soon as possible, and bring me the permission slip from the Dean of Student which allows me to schedule your make-up.

Material:

- **Material on which you will be tested on the Final:**

Retesting on: Chapter 4: Sections 4.1, 4.2, 4.3, 4.5, 4.6; Chapter 5: Sections 5.1

First time testing on: Chapter 5: Section 5.2, 5.3; Chapter 6: Sections 6.1, 6.2, 6.4

- Background material you were tested on in Exam 1 and Exam 2 that you need to know in order to be able to answer questions on the final:

Chapter 1, Sections 1.3, 1.4, 1.7 (**on Exam 1 Review Sheet**)

Chapter 2, Sections: 2.1, 2.2, 2.3, Chapter 3, Sections: 3.1, 3.2 (**on Exam 2 Review Sheet**)

- Homework points = 10 points (1 point for each first-time tested section + 5 points for 3 group-works)
- Final exam points = 90 points
- You may bring a Scientific Calculator (but not a programmable or symbolic calculator)
- You may not bring any notes or handouts

The exam will cover the material from the sections mentioned above, that we discussed in class and studied in the homework assignments. Suggested practice exercises: THE PRACTICE PROBLEMS at the end of each section, and exercises in the same groupings as those assigned as homework problems.

Section by section highlights of the material you should master:

Chapter 4

Section 4.1

Definitions: vector space, subspace of a vector space, a subspace spanned by a set of vectors

Theorems: Theorem 1 (Spanning Set Theorem, page 221)

Skills: determine if a set with addition and scalar multiplication is a vector space, determine if a set of vectors span \mathbb{R}^n , determine if a set is a subspace

Section 4.2

Definitions: The null space of a matrix, Null A; the column space of a matrix, Col A (both descriptions); kernel and range of a linear transformation

Theorems: Theorems 2, 3 (Null A, and Col A are subspaces, pages 227, 229), and highlighted remark on page 230

Skills: Determine if a vector is in Null A or Col A, find a non-zero vector in Null A or Col A, find a spanning set for Null A or Col A

Section 4.3

Definitions: linearly independent and dependent vectors in a vector space, basis of a vector space

Theorems: Theorem 4 (Characterization of Linearly Independent Vectors, page 237), Theorem 5 (The Spanning Set Theorem, page 239), Theorem 6 (Basis for Col A, page 241),

Skills: determine if a set is a basis of a subspace, find a basis for Null A, Col A and other subspaces

Section 4.5

Definitions: finite dimensional vector space, infinite dimensional vector spaces, dimension of a vector space

Theorems: Theorem 9, 10, 11 (Number of Elements in an Independent Set, or a Basis of a Space or Subspace, pages 256, 257, 259), Theorem 12 (The Basis Theorem, page 259), highlighted remark on page 260

Skills: find the dimensions of Null A, Col A and other subspaces, dimension of \mathbb{R}^n and all subspaces of \mathbb{R}^n , geometric meaning of subspaces of \mathbb{R}^n of dimensions 0, 1, 2, and 3.

Section 4.6

Definitions: the row space of a matrix, Row A; the rank of a matrix, rank A

Theorems: Theorem 13 (Basis for Row A, page 263), Theorem 14 (The Rank Theorem, page 265), Theorem (The Invertible Matrix Theorem (continued), page 267)

Skills: find the dimensions and bases for Null A, Col A, Row A, $\text{Col } A^T$ and other subspaces, determine the rank of a matrix, use the Rank Theorem

Chapter 5

Section 5.1

Definitions: eigenvector, eigenvalue, eigenspace

Theorems: Theorem 1 (Eigenvalues of a Triangular Matrix, page 306), Theorem 2 (Eigenvectors of Distinct Eigenvalues, page 307). The remarks following Example 5, page 306: When is 0 an eigenvalue of a matrix

Skills: determine if a number (respectively, a vector) is an eigenvalue (respectively, an eigenvector) of a matrix, find the eigenvalues of a triangular matrix, find a basis for an eigenspace

Section 5.2

Definitions: the characteristic polynomial and equation of a matrix, multiplicity of an eigenvalue, similar matrices

Theorems: Theorem (The Invertible Matrix Theorem (continued), page 312), the highlighted paragraph before Example 3, on page 313, Theorem 3 (properties of determinants, page 313), Theorem 4 (Eigenvalues of Similar Matrices, page 315)

Skills: find the characteristic equation of matrices, find the eigenvalues and their multiplicities of 2×2 and some 3×3 matrices using the characteristic equation, find the eigenvalues and their multiplicities of triangular matrices

Section 5.3

Definitions: diagonalizable matrix

Theorems: Theorem 5 (The Diagonalization Theorem, page 320), Theorems 6 and 7 (Conditions for a Matrix to be Diagonalizable, pages 323, 324)

Skills: decide if a 2x2 or 3x3 matrix is diagonalizable, if A is diagonalizable find P and D such that $A = PDP^{-1}$, show how to compute high powers of diagonalizable matrices

Chapter 6

Section 6.1

Definitions: dot product of vectors, length of a vector, distance between two vectors, orthogonal vectors, unit vector, normalization of a vector, W^\perp the orthogonal complement of a subspace W .

Theorems: Theorem 1 (Properties of Dot Product, page 376), Theorem 2 (The Pythagorean Theorem, page 380, know the proof), highlighted remark after Example 6 (page 381).

Skills: compute dot products, compute length of a vector, compute the distance between two vectors, normalize a vector, decide when two vectors are orthogonal, decide if a vector is orthogonal to a subspace, find W^\perp for selected subspaces W .

Section 6.2

Definitions: orthogonal set of vectors, orthogonal basis, orthonormal set of vectors, orthonormal basis, (orthogonal) projection of a vector on a line or on another vector

Theorems: Theorem 4 (Linear Independence of Orthogonal Vectors, page 384)

Skills: check a set of vectors for orthogonality or orthonormality, compute the projection of a vector onto a line or onto another vector, decompose a vector into a sum of two vectors, one in the direction of \mathbf{u} and another orthogonal to \mathbf{u} .

Section 6.4

Theorems: Theorem 11 (The Gram-Schmidt Process, page 404, memorize the formulas)

Skills: use the Gram-Schmidt process to produce an orthogonal basis from a given basis (consisting of two or three vectors)