Exam 2 Guidelines: Material and Review Suggestions

Date and place: Thursday, March 19, in class
Additional Office Hours Before Exam: Wednesday, March 18, 12:00 – 1:00
Policies: No MAKE-UPS.

This is a one-hour exam, but all students may stay for as long as they need to finish the exam.

Material:

● Chapter 1, Sections: Exam 1 material retested: 1.3, 1.4, 1.7; new material: 1.8, 1.9
  Chapter 2, Sections: 2.1, 2.2, 2.3
  Chapter 3, Sections: 3.1, 3.2
● Homework points total = 10 points (1 point for each new section, 3 points for the two group-works)
● Exam 2 total points = 90 points
● You may bring a Scientific Calculator (but not a programmable or symbolic calculator)
● You may not bring any notes or handouts

The exam will cover the material from the sections mentioned above, that we discussed in class and studied in the homework assignments. Suggested practice exercises: THE PRACTICE PROBLEMS at the end of each section, and exercises in the same groupings as those assigned as homework problems.

Section by section highlights you should master:

Chapter 1

Section 1.3
Definitions: Vectors, Linear combination of vectors, \( \text{Span}\{\mathbf{u}\} \) and \( \text{Span}\{\mathbf{u}, \mathbf{v}\} \) algebraic and geometric interpretations in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \), \( \text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\} \)
Skills: Add, subtract, and multiply vectors by scalars (algebraic and geometric interpretations), Determine when a vector is in the subset spanned by specified vectors, Exhibit a vector as a linear combination of specified vectors, Determine whether a specified set of vectors span \( \mathbb{R}^m \)

Section 1.4
Definitions: \( \mathbf{A}\mathbf{x} \), matrix vector product
Theorems: Theorem 3 (equality of the solutions in the three ways of expressing a system of linear equations, page 42), Theorem 4 (when do columns of \( \mathbf{A} \) span \( \mathbb{R}^m \), page 43), Theorem 5 (properties of matrix-vector multiplication, page 45)
Skills: Determine whether the columns of an \( m \times n \) matrix span \( \mathbb{R}^m \), Compute \( \mathbf{A}\mathbf{x} \) both ways, Be able to switch descriptions between a system of equations, its vector equation, and its matrix equation.

Section 1.7
Definitions: Linearly dependent and linearly independent vectors.
Theorems: Theorems 7, 8, 9 (Properties of linearly independent sets of vectors, pages 68-69).
Skills: Determine whether the columns of a matrix are linearly independent. Determine whether a set of
vectors is linearly independent. Know several methods that can sometimes produce an answer "by inspection", i.e., without much calculation.

**Section 1.8**
Definitions: Linear Transformation, Matrix Transformation
Skills: Use linearity of matrix vector multiplication to compute \( A(u+v) \) or \( A(cu) \), and the linearity of a transformation \( T \) to calculate \( T(cu+dv) \). Determine if a specified vector is in the range of a linear transformation.

**Section 1.9**
Definitions: Standard matrix of a linear transformation
Theory: Theorem 10 (existence of a unique standard matrix for a linear transformation, page 83)
Skills: Find the standard matrix of a linear transformation

**Chapter 2**

**Section 2.1**
Definitions: Identity matrix, zero matrix, diagonal of a matrix, triangular matrix, diagonal matrix, matrix multiplication (both ways), power of a matrix, the transpose of a matrix
Theory: Theorem 1, 2, 3 (Properties of operation with matrices, page 108, 113, 115)
Skills: Add, subtract and multiply matrices, multiply a matrix by a scalar, calculate powers and transposes of matrices

**Section 2.2**
Definitions: Inverse of a matrix, invertible matrix
Theory: Theorem 5 (uniqueness of solution of \( Ax = b \) for invertible matrix \( A \), page 120), Theorem 6 (properties of inverses, page 121), Theorem 7 (characterization of invertible matrices, page 123—you need not know the proof)
Skills: Algorithm for finding the inverse of a matrix

**Section 2.3**
Theory: Theorem 8 (The Invertible Matrix Theorem, page 129), Theorem 9 (inverse of a linear transformation, page 131, this is the material in the group-work)
Skills: Use the Invertible Matrix Theorem to decide if a matrix is invertible or not, and employ the invertibility of the matrix to decide spanning and independence properties of its columns, find the inverse of a linear transformation (if it exists)

**Chapter 3**

**Section 3.1**
Definitions: determinant of a square matrix, cofactor, cofactor expansion
Theory: Theorem 1, 2 (cofactor expansion formula, determinants of triangular matrices, page 188, 189)
Skills: Calculate determinants

**Section 3.2**
Theory: Theorem 4 (characterization of invertible matrices by determinants, page 194), Theorem 5, 6 (determinants of transpose and multiplication of matrices, page 196)
Skills: Use determinants to decide independence and spanning properties of vectors, use properties of determinants to simplify calculations of determinants