The Poetry of Prime Numbers

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Abstract

Prime numbers had been objects of fascination, for both mathematicians and artists, since the time of Euclid. This article explores the links between prime numbers and poetry. We start with a selection of poems celebrating the mathematical properties of these enigmatic and unpredictable integers and their impact on those who explore them. We then turn our attention to poetry reflecting cultural aspects associated with prime numbers, from the use of the concept of primality as a metaphor for the mysteries of life and human behavior to the inclusion of specific prime numbers in poems for the symbolic personal and cultural values these numbers acquired through history. We conclude with a sample of poems highlighting the role prime numbers and their properties play in the aesthetics of poetry, in particular their contribution to the structure of poems. We also include references to additional sources of prime number poetry, and a brief discussion on the uses of such poetry in the mathematics classroom.

Mathematics

Martin Gardner starts his article: *Patterns and Primes* [13], with the following statement:

No branch of number theory is more saturated with mystery and elegance than the study of prime numbers: those exasperating, unruly integers that refuse to be divided evenly by any integers except themselves and 1.

It is therefore no wonder that prime numbers appear, not only in mathematics, but also in that other human endeavor that delves into mysteries in search of patterns and elegance— poetry. A poem that captures many of the elements that made prime numbers objects of fascination since the time of Euclid is Helen Spalding's: *Let Us Now Praise Prime Numbers* [16]. British writer and poet Helen Spalding (1920-1991) is herself a mysterious figure, whose life cannot be traced after her last publication in *The London Magazine* in 1961.

Let Us Now Praise Prime Numbers
by Helen Spalding

Let us now praise prime numbers
With our fathers who begat us:
The power, the peculiar glory of prime numbers
Is that nothing begat them,
No ancestors, no factors,
Adams among the multiplied generations.

None can foretell their coming.
Among the ordinal numbers
They do not reserve their seats, arrive unexpected.
Along the lines of cardinals
They rise like surprising pontiffs,
Each absolute, inscrutable, self-elected.

In the beginning where chaos
Ends and zero resolves,
They crowd the foreground prodigal as forest,
But middle distance thins them,
Far distance to infinity
Yields them rare as unreturning comets.

O prime improbable numbers,
Long may formula-hunters
Steam in abstraction, waste to skeleton patience:
Stay non-conformist, nuisance,
Phenomena irreducible
To system, sequence, pattern or explanation.

The first stanza of Spalding's poem alludes to the mathematical result known as *The Fundamental Theorem of Arithmetic*. *The Fundamental Theorem of Arithmetic* states that every positive integer greater than 1 is either a prime number or can be expressed in a unique way as a product of powers of distinct prime numbers. Therefore, the prime numbers are the (multiplicative) building blocks of the integers and consequently, the building blocks of the entire real number system [6]. The second and third stanzas of Spalding's poem refer to the way prime numbers appear among the other numbers. Scattered among them without a discernable pattern, the prime numbers fan out and appear less frequently as the numbers grow larger. Yet, an infinite number of primes exist. Euclid's proof of the infinitude of prime numbers, circa 300 BC, is considered to be one of the most elegant proofs in mathematics. For mathematicians, this proof is a poem. Michael Szpakowski did justice to Euclid's proof by setting it to music in: *Proof, a Short Opera* [35].

The last stanza of Spalding's poem touches on one of the deep mysteries associated with prime numbers— our inability to pin them down with a formula. Prime numbers smaller than a given number N can be found through an ancient technique called *The Sieve of Eratosthenes* (276–195 BC). If N is not very large, the sifting consists of a simple divisibility testing and the systematic deletion of all the proper multiples of the prime numbers up to the largest prime smaller than the square-root of N. For N = 100, for example, the deletion leaves in the sieve the first twenty-five prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

In this small sample it is difficult to see the thinning of the number of primes and the lack of pattern in their distribution that is apparent as the numbers increase. Although many techniques were invented since Eratosthenes to "catch" prime numbers, no formula has been found that covers them all. In particular, it is notoriously difficult to produce very large primes. Neither has a pattern been found to predict their distribution within a given interval of numbers. An advance in resolving these questions will not only be intellectually and aesthetically pleasing, but will also have applications to public-key cryptography. The Clay Mathematics Institute, founded in 1998, listed the seven most important open problems in mathematics, the *Millennium Prize Problems* [4]. A solution to any of the millennium prize problems will be awarded a million dollars upon publication. One of the millennium prize problems, the Riemann Hypothesis, proposed by Bernhard Riemann (1826-1866), celebrated its 150^{th} anniversary in 2010. The Riemann Hypothesis is a conjecture about the zeros of the Riemann zeta function. It is considered to be the most important open problem in pure mathematics, whose solution will advance our knowledge of the distribution of prime numbers. Below is a fragment from Tom Apostol's Riemann Hypothesis poem: *Where Are the Zeros of Zeta of s?* [18]:

***from*: Where are the Zeros of Zeta of s?**
by Tom Apostol

Where are the zeros of zeta of s?
G.F.B. Riemann has made a good guess;
They're all on the critical line, saith he,
And their density's one over 2pi log t.

This statement of Riemann's has been like a trigger
And many good men, with vim and with vigor,
Have attempted to find, with mathematical rigor,
What happens to zeta as mod t gets bigger.

There are many other open questions involving prime numbers. Some made their way into poetry. For example, Jason Earls' poem: *Twin Primes* [11] and Martin Huxley's limericks: *Rapport Sur la Conférence* [23], involve open problems and results concerning prime numbers. A poem expressing the spell cast by prime numbers on those who study them was written by the Number Theorist Kazuya Kato [26].

Prime Numbers
by Kazuya Kato

The song of prime numbers sounds Tonnkarari,
We can hear if we keep our ears open,
We can hear their joyful song.

The song of prime numbers sounds Chinnkarari,
Prime numbers sing together in harmony
The song of love in the land of prime numbers.

The song of prime numbers sounds Ponnporori,
Prime numbers are seeing dreams,
They sing the dreams for the tomorrow.

Cultural Connections

In the book, *Prime Numbers: A Computational Perspective* [7], R. Crandall and C. Pomerance, say: "Just as practical applications of prime numbers have emerged in the cryptographic, statistical, and other computational fields, there are likewise applications in such disparate domains as engineering, physics, … and biology … beyond the scientific connections there are what may be called the 'cultural' connections."

The cultural connections referred to in this passage manifest themselves in poetry in a variety of ways. The concept of primality appears in poems as a metaphor for the intoxicating mysteries of life and human behavior. Cathy Colman's poem, *Borrowed Dress* [5] uses primality as metaphor. Below is another example of this cultural phenomenon, poet and journalist Jim Mele's poem, *Prime Numbers* [30].

Prime Numbers
by Jim Mele

Prime numbers,
I remember them
like drinks
following complicated folk laws.
Out in California
a friend visits a pebble
beach, indivisible
in this uncertain life.

The depth of the cultural connection between prime numbers and poetry reveals itself more clearly when examining the phenomenon of the inclusion of specific prime numbers in poems. There is a strong affinity between numbers and words whose roots go back to the invention of the alphabetic writing by the Phoenicians in the 2nd millennium BC, when numbers came to be denoted by letters of the alphabet [24]. In ancient poetry, especially in the domain of magic, mysticism and divination, every word acquired the number value of the sum of its letters and every number partook in the symbolic values of one or more words in whose spelling it appeared. The historian of mathematics D. E. Smith mentioned [33] that 3 and 7 "were chief among mystic numbers in all times and among all people" and explained the reasons as lying "in the fact that 3 and 7 are the first prime numbers,—odd, unfactorable, unconnected with any common radix, possessed of various peculiar properties…" In other words, the reason 3 and 7 acquired a special significance in antiquity is precisely because of their primality. Vestiges of ancient values and significance combined with later layers of cultural, sociological and historical meaning, make specific prime numbers evoke powerful images and emotions. Poets include prime numbers in their poems imbued with both personal meaning and collective cultural value. Prime numbers appear in poetry either in isolation or as part of a collection of numbers calculated to achieve a particular effect. Due to space limitations, we include here only a small selection of poems featuring the prime number 7.

Beginning when the world was created and simultaneously the count of time was started, number 7 appears in the first poem of *Genesis,* the first book of *The Bible* [2]:

***from*: Genesis**

> Thus the heavens and the earth were finished, and all the host of them.
> And on the seventh day God ended his work which he had made; and he rested on the seventh day from all his work which he had made.
> And God blessed the seventh day, and sanctified it: because that in it he had rested from all his work which God created and made.

Number 7 is mentioned in many ancient sources, both religious and secular. It appears in *The Epic of Gilgamesh* [14]— the earliest known work of literature (circa 27th century BC), as well as in the Bible, the New Testament, and the Koran. Another ancient verse is the story of the 7 years drought in Egypt (circa 300 AD) inscribed in hieroglyphic form on the stella called *The Famine Stella on Sehel Island* [28]. These appearances connect the number 7 to an indivisible cycle of time in which an event or situation is brought to a successful or disastrous conclusion. Modern poems Lisel Mueller's: *Reasons for Numbers* [31] and Dannie Abse's: *How I Won the Raffle* [1] reflect the ancient layers of history and mystery that number 7 carried with it into the present:

***from*: Reasons for Numbers**
by Lisel Mueller

7

Because luck
is always odd
and the division
of history
into lean and fat
                    years
mysterious

***from*: How I Won the Raffle**
by Dannie Abse

I chose 7 because those ten men used to dance
around the new grave seven times.

Also because of the pyramids of Egypt;
the hanging gardens of Babylon;
Diana's Temple at Ephesus;
the great statue of Zeus at Athens;
the Mausoleum at Halicarnassus;
the Colossus of Rhodes;
and the lighthouse of Alexandria.

Langston Hughes' poem, *Addition [1]* [22], below, features the number 7 in a somewhat unusual way. *Addition [1]* is a minimalist poem, in the sense that it conveys a message with a minimal amount of words. It also has a shape, a typographical pattern, which conveys meaning. Such poems are called concrete or pattern poems. A poem's impact on the reader often lies in a personal interpretation. I will, nevertheless, hazard the guess that the 7s in *Addition [1]* are "lucky numbers" and the poem comments on the meaning of the addition of "love" to "luck."

Addition [1]
by Langston Hughes

7 x 7 + love =
An amount
Infinitely above:
7 x 7 – love.

Other poets invoke the number 7 for a variety of reasons, some compelling enough to include the number in the title of the poem. To mention a few: William Wordsworth's: *We Are Seven* [39], Frederico Garcia Lorca's: *Song of the Seven Maidens* [29], Attila Jozsef's: *The Last of Seven* [25].

Below is a fragment from Lewis Carroll's classic verse, *The Hunting of the Snark* [16], which mentions number 7 in company of other numbers for an amusing mathematical effect. Do the math!

***from*: The Hunting of the Snark**
by Lewis Carroll

"Taking Three as the subject to reason about—
    A convenient number to state—
We add Seven, and Ten, and then multiply out
    By One Thousand diminished by Eight.

"The result we proceed to divide, as you see,
    By Nine Hundred and Ninety and Two:
Then subtract Seventeen, and the answer must be
    Exactly and perfectly true.

Small prime numbers also appear in Pablo Neruda's: *Ode to Numbers* [16], Amy Uyematsu's: *The Invention of Mathematics* [37], Robert Creeley's: *Numbers* [8], Sarah Glaz's: *I am a number* [19], Wallace Stevens': *Thirteen Ways of Looking at a Blackbird* [34], Kenneth Koch's: *The Magic of Numbers* [27] and others. The occurrence of large prime numbers in poetry follows the pattern of their occurrence among the numbers— it thins out as they increase. A poem featuring a very large prime number appears in the next section.

Aesthetics and Structure

A rare occurrence is the appearance of a prime number in a poem due to its visual appeal. Yet, this is the reason for the appearance of number 5 in William C. William's imagist poem, *The Great Figure* [38]. William's poem made others aware of the aesthetic quality of the great figure 5. American artist Charles Demuth's (1883–1935) painting, *I Saw the Figure 5 in Gold* [9], appearing to the right of William's poem below, was inspired by this poem, as is the multimedia version of the poem at: Poems That Go [12].

The Great Figure
by Williams Carlos Williams

Among the rain
and lights
I saw the figure 5
in gold
on a red
firetruck
moving
tense
unheeded
to gong clangs
siren howls
and wheels rumbling
through the dark city.

Figure 1. Charles Demuth, *I Saw the Figure 5 in Gold*
www.metmuseum.org

Although images of numbers are not often associated with the aesthetics of poetry, numbers do play an important role in the aesthetic of poetry through their contribution to poems' structure. A poem's musicality and the mood it evokes depend, not only on words, but also on structural elements that can be measured. Formal poetry relies on counting to achieve desired forms. The count includes meter, rhyme, line length, number of lines in a stanza, number of stanzas in the poem, and more. A certain amount of mathematical calculations, either formal or intuitive, are involved even in free verse. Prime numbers appear among these counted patterns. More information on the role counting plays in poetic structure is found in [3] and [21]. In this section we will bring two examples of non-traditional poem structures that use the mathematical properties of prime numbers.

The first example involves the use of *The Fundamental Theorem of Arithmetic*, mentioned in the first section of this article, for the purpose of constructing a poem. To construct a poem using this theorem first decide on the length of your poem and number the poem's lines consecutively from bottom to top: 2, 3, 4, 5…. Then choose a word that stands for multiplication and a word that stands for exponentiation. The next step is to write the lines corresponding to prime numbers. Each line that acquired a prime number is a building block of the other lines much like the prime numbers build the positive integers.

Below is my poem, *13 January 2009* [20], which was constructed using this approach. Here is how the poem was built: Line 2 is: *Anuk is dying,* line 3 is: *The white of winter,* the word "in" stands for multiplication, and the word "for" stands for exponentiation. To construct, for example, line 12, we write $12 = 2^2$ x 3, and replace the number 2 by line 2, the number 3 by line 3, multiplication by "in" and exponentiation by "for." This makes line 12: *Anuk is dying for Anuk is dying in the white of winter.*

13 January 2009
by Sarah Glaz

$12=2^2$x3	Anuk is dying for Anuk is dying in the white of winter
11	The coldest month
10=2x5	Anuk is dying in the falling snow
$9=3^2$	The white of winter for Anuk is dying
$8=2^3$	Anuk is dying for the white of winter
7	The drift of time
6=2x3	Anuk is dying in the white of winter
5	The falling snow
$4=2^2$	Anuk is dying for Anuk is dying
3	The white of winter
2	Anuk is dying
1	.

Reading the lines from the bottom upward renders another poem. The echo created by the repetition of the prime numbered lines evokes an elegiac mood. All poems I know employing this technique express feelings of sadness. The first poem to use this structure was Carl Andre's poem, *On the Sadness* [16]. JoAnne Growney's: *We Are the Final Ones* [21], is another poem using this structure.

The second example involves the aesthetic manipulation of very large primes. Below is Jason Earls' concrete prime poem, *Lighght Prime* [10], based on Aram Saroyan's controversial poem, *Lighght* [32].

Lighght Prime
by Jason Earls

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11111111111111111111111111111111111111111111111111111111111111111111111111
10000000000000000000000000000000000000000000000000000000000000000000000001
10000000000000000000000000000000000000000000000000000000000000000000000001
10008800000000888888000888888000880000880008888880008800008800888888880001
10008800000000008800008800008800880000880088000088008800008800000880000001
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10008888888800888888000888888000880000880008888880008800008800000880000001
10000000000000000000000000000000000000000000000000000000000000000000000001
10000000000000000000000000000000000000000000000000000000000000000000000001
11111111111111111111111111111111111111111111111111111111111111111111111111
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x 10^{1280} – 1

Multiplying the linear arrangement of the digits in the matrix spelling "lighght" by 10^{1280} and then subtracting 1 yields a very large prime number. The verification that this number is indeed prime involves the use of a computer program. Earls' book, *The Lowbrow Experimental Mathematician* [10], includes additional information on this poetic form and more concrete prime poems.

We end this section with a mention of another technique for constructing poems involving, perhaps accidentally, the prime number 7. This technique, called the *N* + 7 *algorithm*, was invented by the French Oulipian poet Jean Lescure. Oulipo— Ouvroir de Litterature Potentielle (Workshop of Potential Literature) is a literary movement founded by Raymond Queneau in 1960. Its purpose is to create literary works using constrained writing. Many of the constraints invented by Oulipo members are mathematical. The *N + 7* algorithm is a procedure that replaces each noun in a given poem with the 7^{th} noun that follows in a specified dictionary. Mathematically, this is a function on the set of nouns that "translates" each noun by 7 units. The results are amusing. With the advent of computers it became easy to extend this algorithm to numbers other than 7. You can try this technique using your favorite poem at, *The N + 7 Machine* [36].

Concluding Remarks

Mathematical poetry appears in the mathematics classroom through the ages, and at all mathematical levels. The nature and frequency of its use as a tool for teaching mathematics fluctuates to reflect technological advances and changing attitudes to mathematics education. But regardless of the specific reasons for the inclusion of a poem in a class, the power of poetry to engage attention and enhance memory is always an underlying presence. In addition to enrichment of pedagogy through engagement and enhancement of retention, poetry is often used in the mathematics classroom to shape course content, to facilitate integration of material, and to ease the transition from theory to applications. One of the ways in which mathematical poetry shapes course content is by focusing attention on a particular aspect of the material taught in class, and acting as a springboard to initiate class-wide or small group discussions, assignments, or projects based on the poem's content. Judicious choice of poems and careful project construction often result in additional pedagogical benefits, such as better integration of material and easier transition to its applications. A different type of poetry project, with similar aims and results, requires students to compose their own poems about mathematical techniques or concepts. Examples of the use of poetry projects in college mathematics classes, a survey of the efforts made by educators in this direction, and an extensive bibliography, may be found in [15, 17, 18]. Recently, I have been experimenting in my sophomore level mathematics classes with handing out poems on topics that the students have not yet learned, but which touch marginally on the classroom material. The intention is to pique curiosity and motivate the students to take the next step in their mathematical education, enroll in a more advanced mathematics course. I have not yet conducted a controlled study to assess the results of this motivation technique, but I can report that the students in my classes seemed to enjoy my poetic experiments and a number of them actually enrolled in more advanced mathematics classes than they originally intended. The poems appearing in this article may be used in the mathematics classroom to enhance pedagogy or to motivate students to take a course in Number Theory or Abstract Algebra. Most of all, I hope that the poems presented here will inspire mathematicians and educators to write their own mathematical poems, and to experiment with innovative uses of poetry in their classes.

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