

The Enigmatic Number e : A History in Verse and Its Uses in the Mathematics Classroom

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Introduction

In this article we present a history of e in verse—an annotated poem: *The Enigmatic Number e* . The annotation consists of hyperlinks leading to biographies of the mathematicians appearing in the poem, and to explanations of the mathematical notions and ideas presented in the poem. The intention is to celebrate the history of this venerable number in verse, and to put the mathematical ideas connected with it in historical and artistic context. The poem may also be used by educators in any mathematics course in which the number e appears, and those are as varied as e 's multifaceted history. The sections following the poem provide suggestions and resources for the use of the poem as a pedagogical tool in a variety of mathematics courses. They also place these suggestions in the context of other efforts made by educators in this direction by briefly outlining the uses of historical mathematical poems for teaching mathematics at high-school and college level.

Historical Background

The number e is a newcomer to the mathematical pantheon of numbers denoted by letters: it made several indirect appearances in the 17th and 18th centuries, and acquired its letter designation only in 1731. Our history of e starts with John Napier (1550-1617) who defined logarithms through a process called dynamical analogy [1]. Napier aimed to simplify multiplication (and in the same time also simplify division and exponentiation), by finding a model which transforms multiplication into addition. He proceeded to construct such a model using two lines, each of which contains one moving point, and figuring out a correspondence between various line segments generated by the moving points. This could be translated into a two-column numerical table where the numbers in one column are in one-to-one correspondence with the numbers in the second column, and multiplication of two numbers in one column is equivalent to addition of the corresponding numbers in the second column (modulo a factor of 10^7). In other words, Napier came up with a model that shares with today's logarithms the property of transforming multiplication into addition. The model is



John Napier (1550-1617)

almost equivalent to what we call logarithm today, namely: $y = \log_b x$ if and only if $b^y = x$.

Napier's definition did not use bases or algebraic equations. Algebra was not advanced enough in Napier's time to allow for our modern definition. Logarithmic tables were constructed, even tables very close to natural logarithmic tables, but the base, e , did not make a direct appearance till about a hundred years later. Gottfried Leibniz (1646-1716), in his work on Calculus, identified e as a constant, but labeled it b . As with many other concepts, it was Leonhard Euler (1707-1783) who gave the constant its letter designation, e , and discovered many of its remarkable properties. Euler's discoveries cast new light on the previous work, bringing out e 's relevance to a host of results and applications. More details on the history of e may be found in the next section and in references [1, 4, 7, 12, 13, 14, 26, 27, 28, 30, 31, 33, 34]. In particular, it is interesting to note that a model simplifying multiplication by transforming it into addition was discovered at about the same time as Napier's logarithm by another mathematician, Jost Bürgi (1552-1632). See, for example, [30] for some of the controversies regarding priorities for the discovery and the early calculations of logarithms.



Leonhard Euler (1707-1783)

Compared with the number π that appeared in print as early as 550 BC (The Hebrew Bible, I Kings vii. 23) and whose accuracy of digits following the decimal point traces the history of ancient mathematics [3], e seems to have little to boast about. In addition, the geometric definition of π is easily accessible to any literate person, while e 's abstract meaning requires more advanced mathematical knowledge even for basic comprehension. For these reasons, π became part of the popular culture: songs, poems, movies, cartoons, websites, and even an annual holiday (Pi-Day, on March 14), are dedicated to it (see, for example, [15]), while e still awaits public recognition.

Among mathematicians, however, e is considered to be one of the most important numbers in mathematics [14], along with π , i , 0 and 1 , all of which are linked in the famous and mysterious Euler Identity, $e^{i\pi} + 1 = 0$. Moreover, there is a special fascination to e 's varied and unexpected appearances at the core of several important areas of modern mathematics. If π 's long history traces the ancient development of mathematics, e 's shorter history traces the birth of modern mathematics. And it is a captivating history, complete with eccentric personalities, spectacular mathematical results, and still unsolved conjectures—a history worth celebrating and sharing with students.

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The Annotated Poem

The Enigmatic Number e

by Sarah Glaz

It ambushed [Napier](#) at Gartness,
like a swashbuckling pirate
leaping from the base.
He felt its power, but never realized its nature.
 e 's first appearance in disguise—a tabular array
of values of [ln](#), was logged in an appendix
to [Napier](#)'s posthumous publication.
[Oughtred](#), inventor of the [circular slide rule](#),
still ignorant of e 's true role,
performed the calculations.

A hundred thirteen years the hit and run goes on.
There and not there—elusive e ,
escape artist and trickster,
weaves in and out of minds and computations:
[Saint-Vincent](#) caught a glimpse of it under [rectangular hyperbolas](#);
[Huygens](#) mistook its rising trace for logarithmic curve;
[Nicolaus Mercator](#) described its log as natural
without accounting for its base;
[Jacob Bernoulli](#), compounding interest continuously,
came close, yet failed to recognize its face;
and [Leibniz](#) grasped it hiding in the maze of calculus,
natural basis for comprehending change—but
misidentified as b .

The name was first recorded in a [letter](#)
[Euler](#) sent [Goldbach](#) in November 1731:
"e denontat hic numerum, cujus logarithmus hyperbolicus est=1."
Since a was taken, and [Euler](#)
was partial to vowels,
 e rushed to make a claim—the next in line.

We sometimes call e [Euler's](#) Number: he knew
 e in its infancy as 2.718281828459045235.

On Wednesday, 6th of May, 2009,
 e revealed itself to [Kondo and Pagliarulo](#),
digit by digit, to 200,000,000,000 decimal places.

It found a new digital game to play.

In retrospect, following [Euler's](#) naming,
 e lifted its black mask and showed its [limit](#):

$$e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

[Bernoulli's](#) [compounded interest](#) for an investment of one.

Its reciprocal gave [Bernoulli](#) many [trials](#),
from gambling at the slot machines to [deranged parties](#)
where nameless gentlemen check hats with butlers at the door,
and when they leave, e 's reciprocal hands each a stranger's hat.

In gratitude to [Euler](#), e showed a serious side,
[infinite sum](#) representation:

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$

For [Euler's](#) eyes alone, e fanned the peacock tail of
 $\frac{e-1}{2}$'s [continued fraction](#) expansion,
displaying patterns that confirmed
its own [irrationality](#).

A century has passed till e —through [Hermite's](#) pen,
was proved to be a [transcendental number](#).
But to this day it teases us with
speculations about e^e .

e 's abstract beauty casts a glow on [Euler's Identity](#):

$$e^{i\pi} + 1 = 0$$

the elegant, mysterious equation,
where waltzing arm in arm with i and π ,
 e flirts with [complex numbers](#) and [roots of unity](#).

We meet e nowadays in functional high places
of [Calculus](#), [Differential Equations](#), [Probability](#), [Number Theory](#),
and other ancient realms:

$$y = e^x$$

e is the base of the unique [exponential function](#)
whose [derivative](#) is equal to itself.
The more things change the more they stay the same.
 e gathers gravitas as solid under [integration](#),

$$\int e^x dx = e^x + c$$

a constant c , is the mere difference;
and often e makes guest appearances in [Taylor series expansions](#).
And now and then e stars in published poetry—
honors and administrative duties multiply with age.

Uses of Historical Poems in the Mathematics Classroom

The links between the history of mathematics, or mathematics-related art, and the teaching of mathematics are complex and difficult to classify. Generally acknowledged is the fact that both history and art can be used to enrich the teaching of mathematics by strengthening students' engagement with the material. The power of a historical anecdote or a poem to engage and charm may lie in the response to the contrast between the abstract and impersonal mathematical idea, and the emotional and aesthetic nature of a piece of related art or a historical tale associated with the idea. The power may also reside in the heightened interest generated by a presentation of mathematical ideas in a broad context, a context that includes historical, social, and artistic dimensions, in addition to the mathematical one. This pedagogical aspect of either history or art in the teaching of mathematics is present, even if the original reasons for their inclusion in the classroom may have been different. In addition to enrichment of pedagogy through engagement, both history and art are often used in the mathematics classroom to shape course content and to enhance learning, retention, and integration of material. The reference section includes a small, but representative, selection of sources: [5, 7, 21, 22, 23, 30, 31] contain discussions, ideas, and classroom resources for the use of history in teaching mathematics, while [2, 6, 8, 9, 18, 19, 20, 25, 29, 32] elaborate on parallel themes for the use of poetry.

The use of a combination of history and poetry in the high-school and college mathematics classroom is not as prevalent as their separate use. One reason may be that many of the historical sources of mathematics that were originally written in verse, like, for example, much of the Middle Ages mathematics written in India, were translated as prose. Still, some historical poems are available in English, and poetry and history were also successfully combined in other ways to enhance the teaching of mathematics.

Poems, or rhymes, have been used for a long time as mnemonics for important numbers, like π and e , or to assist with the memorization of techniques or formulas, like, for example, the formula for finding roots of a quadratic equation. Some of these rhyming mnemonics are of historical vintage (see for example [18, 25, 32]).

An interesting use of the combination of history and poetry appears in [20], where history provides the motivation for the introduction of poetry in an algebra course. The motivating history is the flowering of algebra during the Middle Ages in India with its cultural tradition of recording mathematical results and problems in verse. Some of the most charming mathematical poems come from this tradition. For example, Bhaskara (1114-1185), the best known of medieval Indian mathematicians, wrote an algebra book some believe was intended for the education of his daughter, Lilavati. The book's title is also *Lilavati* (meaning "the beautiful"), and it was written entirely in verse. The translation of *Lilavati* into English is in prose [10], but luckily a few of the poems were translated as verses in other sources (see [18] for three such translations and their sources). Inspired by the mathematical poetry of medieval India, Barbara Jur [20] encouraged her algebra class to compose word-problems in poetry. The results have both mathematical and poetic merit. Jur's motivation was to enrich teaching by engagement, but in articles [2, 29] we find reports of such poetry writing

experiments conducted in Pre-Calculus, Calculus, and Statistics classes, which conclude that poetry writing in mathematics classes strengthens students understanding and integration of the subject matter.

The mathematical poetry of medieval India and the difficulties students have with word-problems in algebra feature in another article describing the use of poetry in a college algebra course [18]. Glaz & Liang [18] used poems from *Lilavati*, and other historical sources, to ease the difficulties students have with the transition between word-problems representing natural phenomena and the corresponding mathematical models—the equations representing the phenomena. The process yielded additional pedagogical benefits, like, for example, the strengthening of students' number sense and mathematical intuition, and the enhancement of retention and integration of the material [18].

In [18] the authors also introduce a poem by Glaz, *Calculus*, [16, 17, 18], which, like *The Enigmatic Number e*, combines biographical details about the mathematicians involved with the history of mathematical ideas. A poem of this kind may be used to enhance course content by acting as a springboard to class-wide or small group discussions, projects, or assignments on the topics presented in the poem. This kind of poetry-related classroom activities and their implications for the learning process are discussed in [9, 18, 25], while historical accounts in prose with suggestions for these kind of classroom activities may be found in [5, 9, 31].

I will conclude this section with a few words about my own experience with the use of historical mathematical poetry. The poetry I use is either old poetry with mathematical content or contemporary poetry that combines mathematics and history in its content. In addition to class discussions, I assign group projects on, or related to, the mathematical content of the poem. As a rule, the more elementary the course the more structured the project. The rationale behind my approach takes into account that students in higher level mathematics courses are academically and mathematically more mature and are therefore capable of extracting benefits from the inclusion of poetry and history in their class with less supervision. In particular, in a freshmen Problem Solving course and in a remedial College Algebra course the projects I assigned had step by step instructions to the mathematics involved along with an introduction describing some of the history of the poems. For samples of projects at this level, the reader may consult reference [18].

The poem *Calculus*, mentioned above, was used in a Calculus class to enliven the review of the material at the end of the chapter on The Fundamental Theorem of Calculus. The project required students to "translate" from words into mathematical statements the lines of the poem describing mathematical concepts and theorems (such as, for example, the definition of the definite integral as a Riemann sum and the Fundamental Theorem of Calculus itself). In a class involving senior mathematics majors I offered Archimedes' *The Cattle Poem* [24] as a handout, with a recommendation that students explore any point that arouses curiosity and an invitation to discuss their explorations with me. In general, students reacted to the poems and the projects with surprise and enjoyment—sometimes surprise at their unexpected enjoyment from a practice that is not common in the discipline.

Using the Poem, *The Enigmatic Number e* , in the Mathematics Classroom

The poem *The Enigmatic Number e* may be used in the mathematics classroom to enrich pedagogy or course content, in any of the number of ways discussed in the previous section. In particular, the poem may be offered to students as a classroom handout, or as a link to an electronic posting, in mathematics courses that mention the number e . These courses range from high-school and college Intermediate Algebra, to Pre-Calculus and Calculus courses, to Business Mathematics courses, to more advanced college courses such as Probability, Differential Equations, Abstract Algebra, Complex Variables, and Number Theory, and, of course, History of Mathematics, or courses exploring the connections between mathematics and poetry. The appearances of e mentioned in the poem are sufficiently varied to engage students with the related material taught in the course, as well as to pique students' curiosity about the mathematics that lies ahead—in courses they have not yet taken.

The Enigmatic Number e may also be used to supplement classroom material and generate classroom activities on topics present in its text. The **Links to Resources** section of this article lists the hyperlinks appearing in the poem's text, as well as links to additional online resources, sorted by biographies and topics. Each link leads to a source of information about a particular mathematician or topic which can assist educators with the preparation of supplementary classroom material and activities. The **Biographies** links contain, in addition to biographical information about the mathematicians, historical information about the mathematical work of each mathematician. Contemporary approaches to the mathematics of a topic may be found in the **Topics** links. The links may also be used to drive course content, by combining several related topics into units of study. Below are two examples of such study units. Each unit consists of a number of parts that can also be used on their own or in different combinations with each other.

Study Unit: The Development of Computation

This unit may start with the definition of Napier's logarithm (links: *Napier*, references: [1], [30]), and his early computation devices called Napier's bones and counting board (links: *Napier*, reference: [7]). In particular, the second link of *Napier* contains information for the preparation of classroom activities and homework projects on the use of these calculation devices.

The unit may continue with the development of the slide rule (links: *Oughtred*, *Slide rule*). The top two links of *Slide rule* provide both historical information and ideas for classroom or homework activities on the historical uses of the various slide rules. The third link of *Slide rule* is an online slide rule powered by Mathematica. Asking students to do both manual and online slide rule calculations will provide a transition to the last part of this unit.

The unit may now transition to modern calculating devices (links: *Logarithm and natural logarithm, e*) In particular, this is a good place to introduce students to both calculator and computer software capabilities (for example, EXCEL or Mathematica), including the capability of computing logs with great ease and e to great accuracy. An online calculator is available at: http://www.calculator.com/cales/calc_sci.html. Online computations of e and logs can be carried out at the third link of *Logarithm*

and natural logarithm and the fourth link of e (click on *Programs*, then click on *Online computation of some constants*). One may conclude this unit with a project asking students to conduct an internet search to discover the latest accomplishment in accuracy for the computation of e , and if time permits, also of π . As of the writing of this article, e has been calculated to 200 billion digits after the decimal point (links: e , *first link*).

Study Unit: The Number System

This unit may start with a classical exposition of the real number system (see, for example, reference [6]) supplemented with the topic of continued fractions (links: *Continued fractions*, reference: [28]). Continued fractions are not a part of any standard college course's syllabus. Their introduction will add a new perspective on the representation of numbers and the difference between rational and irrational numbers. An engaging hands-on approach to this subject, complete with ready made activities and homework exercises, may be found at the top link of *Continued fractions* (scroll up all the way to the top of the site). The other links of *Continued fractions* provide additional information to enrich this material with applications and history of the topic.

The unit may continue with the extension of the real number system to complex numbers (links: *Complex numbers, i, roots of unity*). The link *Complex numbers* provides an interesting self-contained short course on complex numbers—their history and their properties. It can be used to supplement or replace a course's textbook as a basis for lesson plans and assignments.

Keeping e as the main focus, the unit may now explore some properties and relations of the three special numbers mentioned in the poem, e , π , and i (links: *Euler, Hermite, Euler Identity, i, π , e , Irrational numbers, Transcendental numbers*). One can start by introducing the concept of a transcendental number as early as the study of roots of polynomials in an Intermediate Algebra course. The second link of *Transcendental numbers* provides a gentle introduction and several examples to the concept. For more advanced courses, proofs of e 's irrationality and transcendentality may be found in the second and third links of e . The first link of *Transcendental numbers* brings up, without proof, the transcendentality of π and the open questions regarding transcendentality of powers (and addition or multiplication) of π and e . The links for *Euler Identity* can be used to introduce this identity and explore some of its implications to the calculations of powers of the three special numbers. One way to involve the students with the material is to send them on an exploration project through these links for the purpose of finding out what is known about a list of powers of the three numbers, in terms of value, irrationality, or transcendentality. The project may request a justification for each answer, whenever possible (for example, when the conclusion comes out from a direct application of Euler's identity) and a history of the answer. Such a list may include e^i , e^π , e^e , i^i , i^π , i^e , π^i , π^π , π^e , and other more complicated powers.

The **Links to Resources** section's 19 biographical links and 43 topics links, and the **References** section's 34 references, may be combined in many ways to make coherent units of study to suit instructor's tastes, time availability, and content of the course they are intended to be used in.

Links to Resources: Biographies and Topics

Biographies

The hyperlinks to biographical information in the poem's text are from the excellent source: J.J. O'Connor and E.F. Robertson, [The MacTutor History of Mathematics Archive](#), University of St Andrew Scotland. Additional biographical and historical links are provided below for variety and pedagogical usefulness.

- Jacob Bernoulli (1654-1705)
http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Bernoulli_Jacob.html
<http://www.daviddarling.info/encyclopedia/B/Bernoulli.html>
- Jost Bürgi (1552-1632)
<http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Burgi.html>
- Leonard Euler (1707-1783)
<http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Euler.html>
<http://plus.maths.org/issue42/features/wilson/>
- Christian Goldbach (1690-1764)
<http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Goldbach.html>
- Euler's correspondence with Goldbach: *Lettre XV, November 25, 1731*
<http://math.dartmouth.edu/~euler/correspondence/letters/OO0729.pdf>
- Charles Hermite (1822-1901)
<http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Hermite.html>
- Christiaan Huygens (1692-1695)
<http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Huygens.html>
- Gottfried Leibniz (1646-1716)
<http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Leibniz.html>
<http://www.gottfriedleibniz.com/>
- Nicolaus Mercator (1620-1687)
http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Mercator_Nicolaus.html
- John Napier (1550-1617)
<http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Napier.html>
<http://plus.maths.org/issue44/features/sangwin/index.html>
<http://www.johnnapier.com/>
- William Oughtred (1574-1660)
<http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Oughtred.html>
http://www.sliderulemuseum.com/REF/Oughtred_Aubrey.htm
- Gregorius Saint-Vincent (1584-1667)
<http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Saint-Vincent.html>
- Brook Taylor (1685-1731)
<http://www-history.mcs.st-andrews.ac.uk/Biographies/Taylor.html>

Topics

Wikipedia (<http://en.wikipedia.org>) is a rich online source of information about most of the topics mentioned in the poem. Unfortunately, Wikipedia's open-editing policy makes the accuracy of the information on its site uncertain. For this reason we provide

links to different online sources. Whenever possible, we have chosen sites where the information is presented with students and educators in mind.

- Bernoulli trials
<http://www.cut-the-knot.org/Probability/BernoulliTrials.shtml>
- Calculus
http://math.mit.edu/~djk/calculus_beginners/chapter01/section02.html
- Complex numbers
<http://www.clarku.edu/~djoyce/complex/>
- Compound interest
<http://plus.maths.org/issue11/features/compound/index.html>
- Continued fractions
<http://www.maths.surrey.ac.uk/hostedsites/R.Knott/Fibonacci/cfINTRO.html#othercfs>
<http://archives.math.utk.edu/articles/atuy1/confrac/index.html>
<http://plus.maths.org/issue11/features/cfractions/>
- Derangements and the Hat-Check Problem
<http://www.wmueller.com/precalculus/e/e6.html>
<http://notaboutapples.wordpress.com/2009/08/03/the-two-decks-of-cards-and-the-drunken-hat-check-girl/>
- Derivative
http://math.mit.edu/~djk/calculus_beginners/chapter04/section02.html
- Differential Equations
http://khanexercises.appspot.com/video?v=-_POEWfygmU
<http://www.math-atlas.org/> (click on Differential Equations)
- e
<http://ja0hxv.calico.jp/pai/ee1value.html> (e to 200 billion digits)
<http://www.mathpages.com/home/kmath400.htm> (irrationality of e)
<http://planetmath.org/encyclopedia/EIsTranscendental.html> (transcendentality of e)
<http://numbers.computation.free.fr/Constants/constants.html> (online computation of e)
- Euler's Identity
<http://www.well.com/~davidu/euler.html>
<http://users.forthnet.gr/ath/kimon/Euler/Euler.htm>
- Exponential function
<http://mathworld.wolfram.com/ExponentialFunction.html>
- i (imaginary unit)
<http://www.clarku.edu/~djoyce/complex/numberi.html>
- Integral
http://math.mit.edu/~djk/calculus_beginners/chapter16/section01.html
- Irrational numbers
http://www.cut-the-knot.org/do_you_know/numbers.shtml#rational
http://www.daviddarling.info/encyclopedia/I/irrational_number.html
- Limit
<http://www.coolmath.com/lesson-whats-a-limit-1.htm>
- Logarithm and natural logarithm (\ln)
<http://mathworld.wolfram.com/Logarithm.html>
<http://www.coolmath.com/algebra/17-exponentials-logarithms/09-special-logarithms-02.htm>
<http://demonstrations.wolfram.com/CalculatingIntegerLogarithmsInDifferentBases/>
- Number Theory

<http://mathworld.wolfram.com/NumberTheory.html>

<http://www.math-atlas.org/> (click on *Number Theory*)

- Pi (π)

http://www-history.mcs.st-andrews.ac.uk/HistTopics/Pi_through_the_ages.html

http://www-history.mcs.st-andrews.ac.uk/HistTopics/Pi_chronology.html

- Probability

<http://www.cut-the-knot.org/Probability/Dictionary.shtml>

<http://www.math-atlas.org/> (click on *Probability and Statistics*)

- Rectangular hyperbola

<http://mathworld.wolfram.com/RectangularHyperbola.html>

- Roots of unity

<http://www.clarku.edu/~djoyce/complex/powers.html>

- Series (infinite sums)

<http://plus.maths.org/issue19/features/infseries/>

- Slide rule

<http://www.sliderule.ca/intro.htm>

<http://www.sliderulemuseum.com/?PHPSESSID=52598164a6abd4a522b5ef680a487335>

<http://demonstrations.wolfram.com/SlideRule/>

- Taylor series expansion

http://www.math.hmc.edu/calculus/tutorials/taylor_s_thm/

<http://mathworld.wolfram.com/TaylorSeries.html>

- Transcendental numbers

http://www.daviddarling.info/encyclopedia/T/transcendental_number.html

http://www.cut-the-knot.org/do_you_know/numbers.shtml#algebraic

References

1. A. Audsley, *The Old Computer Hut, History: More about Napier's original logarithm*, <http://www.oldcomputers.arcula.co.uk/bhist1.htm>
2. Patrick Bahls, *Math and metaphor: Using poetry to teach college mathematics*, *The Writing Across the Curriculum Journal* 20, 75-90, 2009
3. Petr Beckmann, *A History of π* , St. Martin Press, New York, 1971
4. E.T. Bell, *Men of Mathematics*, Simon & Schuster, New York, 1937
5. William P. Berlinghoff, and Fernando Q. Gouvea, *Math Through the Ages, a Gentle History for Teachers and Others*, Oxton House Publishers, Farmington, ME, and MAA, Washington DC, 2004
6. Marcia Birken, and Anne C. Coon, *Discovering Patterns in Mathematics and Poetry*, Editions Rodopi, B.V. Amsterdam-New York, 2008
7. Michael J. Caulfield, *John Napier: his life, his logs, and his bones*, MAA Loci: Convergence, <http://mathdl.maa.org/mathDL/46/?pa=content&sa=viewDocument&nodeId=2331>
8. Marion Deutsche Cohen, *Downloads: Permission to Add (the Math Limericks)* <http://marioncohen.com/>
9. Marion Deutsche Cohen, *Truth and Beauty: A Course in Mathematics and Literature*, preprint
10. Henry Thomas Colebrooke, *A Translation of Lilavati with Notes by Haran Chandra Banerji*, Asian Educational Services, New Delhi, India, 1993

11. Julian Lowell Coolidge, *The Mathematics of Great Amateurs*, Dover Publications, New York, 1963
12. William Dunham, *Euler: The Master of US All*, MAA, Washington DC, 1999
13. Euler's correspondence with Goldbach,
<http://math.dartmouth.edu/~euler/correspondence/correspondents/Goldbach.html>
14. Howard Whitley Eves, *An Introduction to the History of Mathematics*, Holt, Rinehart & Winston, 1969
15. Exploratorium, *Annual Pi Day Celebration Page*,
<http://www.exploratorium.edu/pi/index.html>
16. Sarah Glaz, *Calculus*, Humanistic Mathematics Network Journal 26, 30, 2002
Poem: http://www.math.uconn.edu/~glaz/Calculus_by_Sarah_Glaz.html
17. Sarah Glaz, and JoAnne Growney, *Strange Attractors, Poems of Love and Mathematics*, AK Peters, Wellesley, MA, 2008
18. Sarah Glaz, and Su Liang, *Modeling with poetry in an introductory college algebra course and beyond*, Journal of Mathematics and the Arts 3, 123-133, 2009. Link to preprint:
http://www.math.uconn.edu/~glaz/My_Articles/ModelingWithPoetryinAnIntroAlgebraCourse.JMA09.pdf
19. Jacqueline Jensen, *Math 142 Poems Presented*,
http://www.shsu.edu/~mth_jaj/math142/poems.s05.html
20. Barbara A. Jur, *The Poetry of Mathematics*, Primus 1, 75-80, 1991
21. Victor Katz (editor), *Using History to Teach Mathematics, an International Perspective*, MAA Notes 51, Washington DC, 2000
22. Arthur Knoebel, Reinhard Laubenbacher, Jerry M. Lodder, and David Pengelley, *Mathematical Masterpieces: Further Chronicles by the Explorers*, Springer, New York, 2007.
23. Reinhard Laubenbacher, and David Pengelley, *Mathematical Expeditions: Chronicles by the Explorers*, Springer-Verlag, Berlin, 1998
24. H.W. Lenstra, Jr., *Solving the Pell Equation*, Notices of the American Mathematical Society 49, 182-192, 2002
<http://www.ams.org/notices/200202/fea-lenstra.pdf>
25. Lawrence Mark Lesser, *Sum of songs: Making mathematics less monotone*, The Mathematics Teacher 93, 372-377, 2000
26. Eli Maor, *e: The Story of a Number*, Princeton University Press, Princeton, NJ, 1994
27. William Mueller: *The Number e*,
<http://www.wmueller.com/precalculus/e/e0.html>
28. J.J. O'Connor, and E.F. Robertson, The MacTutor History of Mathematics archive, University of St Andrews, Scotland: *The Number e*,
<http://www-history.mcs.st-andrews.ac.uk/HistTopics/e.html>
29. Cheryl C. Patterson, and J. Wayne Patterson, *Poetry Writing in Quantitative Courses*, Decision Sciences Journal of Innovative Education 7, 233-238, 2009
30. Amy Shell-Gellasch, *Napier's e*, MAA Loci: Convergence,
<http://mathdl.maa.org/mathDL/46/?pa=content&sa=viewDocument&nodeId=3209>
31. Amy Shell-Gellasch (editor), *Hands on History, a Resource for Teaching Mathematics*, MAA Notes 72, Washington, DC, 2007
32. Michael Stueben, with Diane Sandford, *Twenty Years Before the Blackboard*, MAA, Washington DC, 1998

33. Eric Weisstein, Wolfram MathWorld: e , <http://mathworld.wolfram.com/e.html>
34. Wikipedia: e (*mathematical constant*)
[http://en.wikipedia.org/wiki/E_\(mathematical_constant\)](http://en.wikipedia.org/wiki/E_(mathematical_constant))

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